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THE STRATEGIC EFFECTS OF LACK OF TRANSPARENCY IN FORWARD CONTRACTING BY GENERATORS WITH MARKET POWER

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The Strategic Effects of Lack of Transparency in Forward Contracting by Generators with Market Power

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The ability of an electricity generating firm with market power to influence the market price depends strongly on the volume the firm has pre-sold in the forward, or hedge, markets. However, the choice of hedge level may be a strategic decision in itself. In this analysis we show that the profit-maximizing choice of the hedge level depends on the extent to which the hedge price varies with the firm's hedging decision, which relates to the transparency of the forward market. A lack of transparency results in the hedge price being independent of the firm's hedge level. In this case the optimal choice of hedging is an all-or-nothing decision and there is no equilibrium level of hedging in pure strategies. This outcome may explain an observed lack of hedge market liquidities in wholesale electricity markets with substantial market power. We perform the analysis for the monopoly and oligopoly cases and extend it by realistic cost functions and various degrees of competitiveness in the market. These results contribute to the extensive body of research on the price formation and strategic behavior in electricity forward and spot markets, as well as providing implications for transparency initiatives in market design.

Key words: market power, hedge market, liquidity, Nash equilibrium, Cournot oligopoly

1. Introduction

The potential which wholesale electricity markets present for the exercise of market power has created concern for regulators, competition authorities, and policy makers in most jurisdictions ever since the liberalization of this sector first started to spread worldwide in the 1990s. And as a consequence, the topic has motivated an extensive body of theoretical and empirical research. Twomey et al. (2005) provides an excellent survey. Whilst our understanding of spot market conduct has become very detailed, the analytical implications of market power in futures are only partially understood, even though it is generally expected to have a direct impact on wholesale prices. A generator which is hedged to a high level has very little incentive to exercise market power, while an unhedged generator may have a significant influence on the spot market prices (Biggar and Hesamzadeh 2014).

Although regulations for increasing the transparency of spot markets are being widely implemented, this is not the case with forward contracts. Prior to the day-ahead auction, the individual contract positions of market participants are rarely apparent, yet the major theme of analytical research in forward contracting, following Allaz (1992), Allaz and Vila (1993), Green (1999), Brown and Eckert (2017), presumes full observability of each company's hedge position.

A small amount of research has departed from this and considered the lack of transparency in the forward markets. Thus, Bagwell (1995) and Hughes and Kao (1997) suggest that unobservability leads only to a Cournot equilibrium without forward contracting, but this is predicated upon a no-arbitrage condition, with forward and spot prices being equal. However, this assumption is confronted both empirically by the widespread emergence of forward premia in the term structure of forward curves relative to the realized spot prices (Bunn and Chen 2013) and analytically in the formulations that reflect forward contracting as manifesting equilibria between the hedging needs of heterogeneous risk averse market participants (Bessembinder and Lemmon 2002). Further, from a theoretical perspective, with market power and the absence of risk aversion, it is challenging to formulate how the no arbitrage equilibrium condition can be imposed without participants knowing the forward positions of each other. Ferreira et al. (2006) addresses this specific aspect in a more pragmatic way by presuming that arbitrageurs can infer, from the forward prices as reported, the quantities traded forward by the physical players with market power. That is an open question, but presuming this inference is possible, competition between arbitrageurs then leads to forward and spot equality, but at equilibrium prices between the competitive outcome and that of Allaz and Vila under full observability.

Whilst a concentrated market structure may lead to market power in the spot product market, this degree of market power will often be diluted in the forward trading because of the extra volume of trading. The German power market for example, which provides the main reference price for power trading in Europe, had a churn rate of 11 in 2016 (DG Energy 2016) implying that 11x the actual demand for a particular hour was traded through the various forward contracts. This extra liquidity is a mixture of speculation and active re-hedging. Related to this consideration, Mercadal (2015), for example, has shown that market power in the Midwest electricity markets was reduced after the 2011 regulatory change permitting financial participation, whilst Borenstein et al. (2008) argued that the lack of financial arbitrage increased the market power abuse in the California electricity crisis. So it is a plausible conjecture that increasing churn is associated with reduced market power in the forward markets.

Furthermore, whilst spot market power is often analyzed through a one-shot day ahead auction for hourly products, forward trading is a continuously repeated process involving products of increasingly longer delivery periods than an hour, the further forward the contracting occurs. Hourly blocks, daily, weekend, weekly, monthly, quarterly, semi-annual and annual products are actively traded on the power exchanges, and this complex product bundling obscures the simple 1:1 relationship between spot and forward products which is generally adopted by theory. Indeed, for this reason, much of the empirical work on forward premia only links day ahead hourly prices to intra-day hourly spot prices (Borenstein et al. 2008, Longstaff and Wang 2004) or in terms of overnight returns (Fleten et al. 2015), whilst in practice most hedging extends further forward.

In addition, the sequential iterations of forward trading are well known to reduce the extent of market power (Allaz and Vila 1993), but they do not necessarily lead to competitive outcomes in both the forward and spot markets. Ito and Reguant (2016) argue, theoretically and with empirical reference to the Iberian electricity market, that whilst sequential forward trading can be pro-competitive, to the extent that speculators cannot change production, market power in the spot market can still persist. Whilst we do not consider sequential trading per se in our analysis, we draw upon these observations of forward product bundling and sequential effects, as well as the higher liquidity of forward markets to suggest the difficulty of inferring participant contract positions from forward price reporting.

We therefore analyze a stylized variation of market power effects in forward trading namely that the forward contract positions are unobservable and, whilst market power may be exercisable by some generators in the spot market, the forward markets are not so responsive to their quantity decisions. This case is compared to the case with the assumption of full observability in order to highlight the contributions of our analysis compared to the existing theme of research. We assume linear demand, risk neutrality and constant marginal costs and show that there is no forward-spot equilibrium in pure strategies. This arises because the profit function of the generators is U-shaped and the maxima lie at the extremes. We demonstrate that this result holds for both a single dominant firm and a Cournot oligopoly although, as we will see, with a larger number of firms there is a tendency towards the conventional competitive equilibrium in pure strategies. The same analysis also holds for quadratic cost functions for the generating firms.

We suggest that this absence of equilibrium in pure strategies result has been illustrated by outcomes in the South Australian region of the Australian National Electricity Market (NEM), in which a dominant generator appears to have alternated between periods of exercising significant market power to raise prices, followed by periods with lower prices, coupled with a simultaneous drop in liquidity in the forward market. We motivate our theoretical analysis briefly by this case study, although we do not suggest that our analysis is a theoretical model of that particular behavior (other factors were also at play), but rather, the narrative adds to the plausibility of our stylized analysis and the conclusions that follow.

Finally, we extend the basic analysis in this paper to consider more general cases of asymmetric oligopolies using numerical examples. We show that the indications obtained from the closed-form solution of the symmetric oligopoly continue to hold. As an example from a topical theme, we provide numerical results for the effect of increased wind power penetration on the hedging decisions of generators in the context of market power. As in the analysis of Bessembinder and Lemmon (2002) without market power, we see that the skewness in price distribution also affects the optimal hedging decisions of the generators.

The paper proceeds as follows. Next, we provide a motivating example from the South Australian region of the Australian National Electricity Market. Section 3 introduces the theory and main results in the context of a single dominant generator and a symmetric Cournot oligopoly. Section 4 presents numerical results confirming the theoretical findings and extending them to more general cases, including the case of wind penetration in Section 5. Section 6 concludes.

2. The experience in South Australia

This study is partly motivated by anecdotal evidence of the behavior of one particular dominant generator in the South Australian region of Australia's National Electricity Market. Although the NEM market is reasonably competitive at most times, problems can arise when transmission constraints limit flows into a specific region. The South Australian (SA) region of the NEM lies at the extreme western end of the NEM. Although the South Australian region is small (in terms of either energy consumption or generation capacity) relative to the rest of the NEM, transmission constraints into South Australia will occasionally bind, giving scope for certain generators in SA to exercise market power (see Biggar (2011), Hesamzadeh et al. (2011)). The largest generator in the South Australian region is the Torrens Island Power Station (TIPS). At times of high demand in South Australia, when transmission limits into South Australia are binding, the total South Australian demand cannot be met by the sum of other generating units in South Australia and the interconnector flows. At these times Torrens Island Power Station is pivotal and may have material market power.

The acquisition of Torrens Island by AGL was approved by the Australian Competition and Consumer Commission in 2007. At that time there was no evidence that TIPS had been exercising market power and, in any case, it was argued that AGL as the new owner of TIPS would not have an incentive to exercise market power. This argument hinged on the observation that AGL was primarily a retailer, selling to downstream customers at a fixed price, and therefore would have no interest in increasing the wholesale price. This analysis turned out to be wrong. In each of the three subsequent summers, on very hot (high demand) days in South Australia, TIPS allegedly withheld capacity from the market, pushing the wholesale spot price in South Australia close to the wholesale price ceiling (which was, at the time, $10,000/MWh). This allegation was confirmed by AER in its annual report[1](AER 2010).

As an example of the way in which TIPS apparently exercised market power, consider one particular episode. Figure 1 shows the offer curve for TIPS at three different time periods on the 11th of January 2010. This was a hot day in South Australia, with temperatures exceeding 41 degrees Celsius. Throughout the morning, TIPS offered more than 900 MW of capacity at a price less than $300/MWh and the wholesale spot price remained in the range $60-80/MWh. Around 12:30 pm, import constraints in SA started to bind. TIPS responded by withdrawing around 400 MW of capacity from the market (by placing this capacity into very high-priced bands, close to the price ceiling of $10,000/MWh). The wholesale spot price rose rapidly to close to the price ceiling, and remained above $9000/MWh for most of the next six hours. By 7 pm in the evening, TIPS restored the withdrawn capacity to the market, offering close to 900 MW at below $300/MWh. The wholesale spot price returned to around $50/MWh for the remainder of the evening.

[1] In 2010 the Australian Energy Regulator (AER) made the following comments in its annual State of the Energy Market report: "Spot prices in South Australia rose by 20% to $82 per MWh in 2009/10, which was the second highest price for any region since the NEM commenced. This outcome reflects that around 50% of NEM prices above $5000 per MWh in 2009/10 occurred in South Australia [...]. Most of these events were associated with opportunistic bidding by AGL Energy, the region's largest electricity generator. AGL Energy owns the Torrens Island power station, which accounts for around 40% of South Australia's generation capacity. Transmission limits on importing electricity from Victoria mean AGL Energy can, on days of high electricity demand, bid a significant proportion of its capacity at prices around the market cap and drive up the spot price. It adopted this type of bidding strategy during many of South Australia's 47 extreme price events in 2009/10. The events typically occurred on days of extreme weather, which led to high electricity demand and a tight regional supply/demand balance."

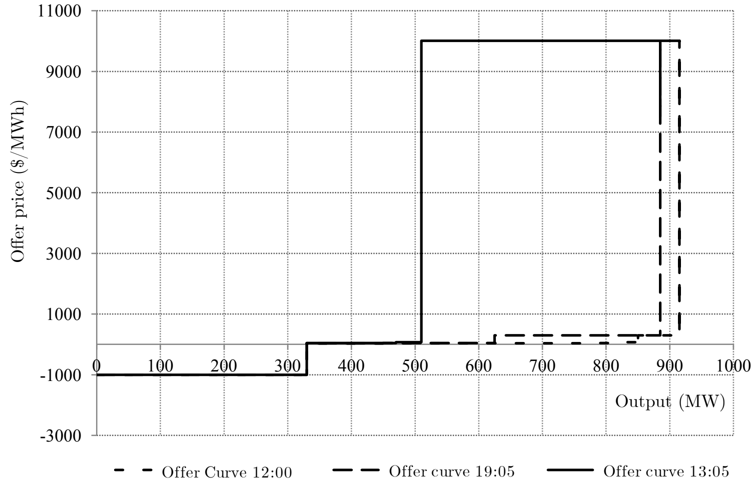


Figure 1 Offer curves of TIPS on 11 January 2011 illustrating possible use of market power to raise the wholesale spot price.

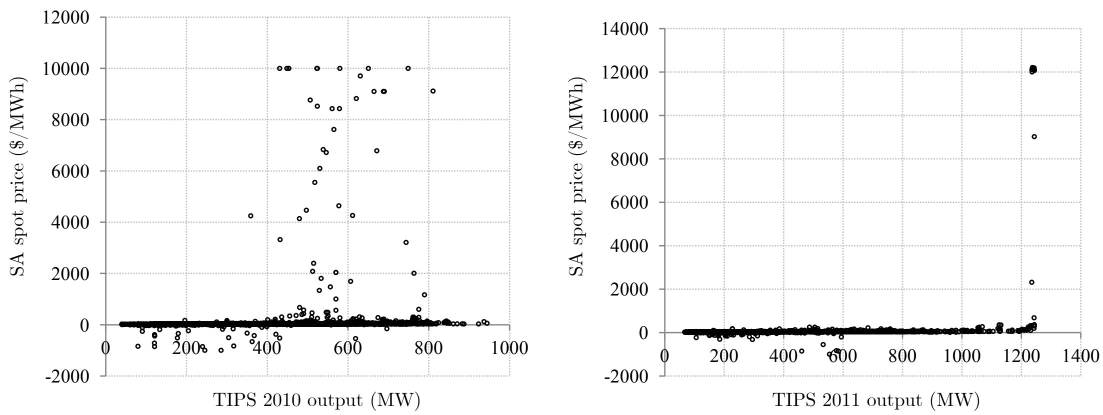


Figure 2 Scatter diagrams of the price-quantity combinations chosen by TIPS during 2010 and 2011.

However, starting in 2011 the behavior of TIPS appeared to switch. Specifically, during 2011 and 2012 TIPS no longer withheld capacity on high-price days, but instead appeared to seek to lower the price at low-demand times. Figure 2 shows two scatter diagrams setting out the price-quantity combinations chosen by TIPS during each half-hour trading interval during the 2010 and 2011 calendar years. As can be seen, during 2010, at times of high prices, TIPS was typically not producing more than around 800 MW and often was producing much less. In 2011, in contrast, at times of very high prices TIPS was consistently producing more than 1200 MW.

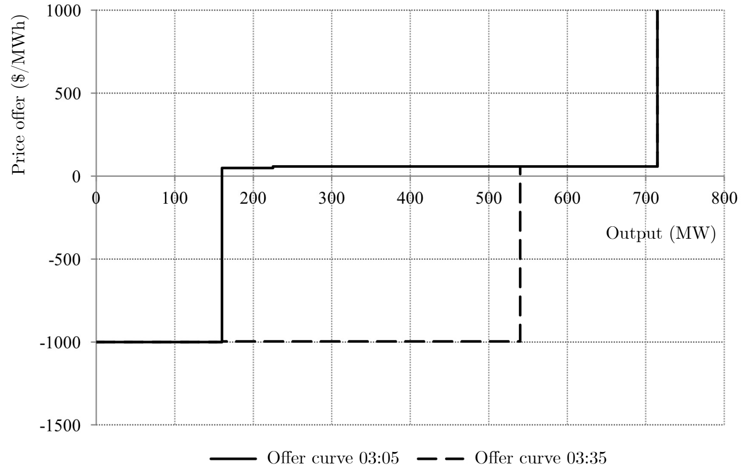


Figure 3 Offer curves of TIPS on 28 June 2012 illustrating possible use of market power to lower the wholesale spot price.

An example of exercising market power to *lower* the spot price occurred during the early hours of 28th June 2012. During June 2012 TIPS would normally reduce its production overnight to a low amount of around 120-160 MW. However, on this occasion at around 3:30 am TIPS increased the amount it offered at the market price floor ($-1000/MWh) from 160 MW up to 540 MW. This increased the production at TIPS up to 540 MW and reduced the wholesale spot price from around $20/MWh to less than $-900/MWh, where it remained for the next two hours. Around 6:30 am TIPS reduced the volume it offered at the price floor. Its output reduced to 220 MW and the wholesale spot price returned to a positive level. The offer curves for TIPS on this occasion at 3:05 am, 3:30 am, and 6:30 am are shown in Figure 3 (AER 2012)[2].

This change in strategic spot market behaviour raises the relevant question regarding the hedging activities of the company. According to industry sources, in 1999/2000 AGL was overwhelmingly dominant as a retailer in South Australia. The owner of the Torrens Island Power Station was able

[2] In its 2012 State of the Energy Market report the AER comments on this as follows: "... all instances of prices below $-100/MWh (including those near the $-1000/MWh market floor) were driven by AGL Energy bidding or rebidding large amounts of capacity to prices near the floor at times of low demand. On several occasions, this effectively shut down other generators (including wind generators). This type of disorderly market activity can have detrimental longer term consequences for market stability and investment."

to hold out for a high price, high volume and medium term hedging contract with the local retailer (AGL). Under this contract Torrens Island was highly hedged and chose not to exercise market power for the subsequent five years. Average wholesale prices in South Australia during this period were moderate. By the time this contract expired around 2006, AGL's market share had eroded and its retail load was smaller. It found that it was able to cover its retail load with hedge contracts purchased from other generators and did not need to purchase hedges from Torrens Island. At this time, faced with low contract prices, the owners of TIPS chose a low hedge level. Following its purchase of TIPS, AGL apparently chose to maintain a largely unhedged position and to exploit opportunities to exercise market power when they arose over the next few years. But, by June 2009 forward prices for electricity in SA were significantly higher than in the other regions of the NEM (AER 2009). In 2010 TIPS appears to have reversed its position and adopted a very high hedge level. Since 2011 TIPS does not appear to have withheld capacity at high-demand times and, indeed, on several occasions appears to have increased the volume it offers to the market at low-demand (and even negative price) times.

This exercise of market power may have had the effect of reducing liquidity in the hedge market. A 2009 report by the Australian Energy Regulator shows that the volume of exchange-based and over-the-counter in South Australian hedge contracts declined significantly after 2006 (AER 2009). A survey of market participants in South Australia in mid-2010 found that the hedge market was illiquid (ACIL 2010). In addition, the forward price for electricity in South Australia has been volatile. In mid-2009 the forward price for a calendar-year 2012 swap was around \$70/MWh (AER 2009). By mid-2010 the price for the same 2012 swap was closer to \$45/MWh (AER 2010).

The above anecdotes indicate that the hedging choices of a dominant generator could be an all or nothing decision. That is, at times of potential market power a dominant generator will either choose to be hedged to a high level or will choose to be unhedged (or even a negative level of hedge cover). Motivated by this example, we revisit and extend some of the basic theory on strategic forward contracting. Under the assumption that the forward market is not transparent we indeed show that the optimal choice of hedging is an all-or-nothing decision and there is no equilibrium level of hedging in pure strategies. This in turn will lead to high and low price regimes in the spot market.

3. Analysis

In this section we derive a closed-form solution to the hedging-decision of a profit-maximizing generating firm in an electricity market. In Section 3.1 we focus on the hedging decision of a single firm. In Section 3.2 we extend the analysis by considering multiple generating firms and a symmetric oligopoly. We start the analysis by considering linear cost functions in Sections 3.1 and 3.2. We further extend the conclusion to the case of quadratic costs in Section 3.3. To keep the problem tractable and to focus on the main results of interest, we will make the following assumptions:

- The hedging contract considered is a swap or contract-for-differences (a hedge contract with a fixed volume of hedge cover determined in advance);
- The generator itself will be assumed to be perfectly reliable.

In the market each generating firm i makes a two-stage decision. In the first stage the firm chooses the level of hedge cover, with knowledge of the range of demand scenrios likely to arise in the future and the prevailing hedge price. In the second stage the firm observes the actual demand and makes a decision on its level of output. This sequence is shown in Figure 4.

The whole setup can be formulated as follows:

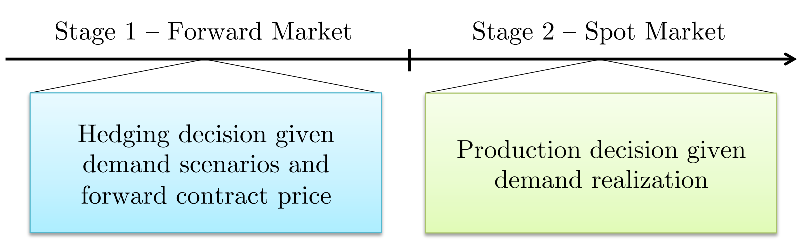


Figure 4 Decision-making of a generating firm.

$$\forall i: \begin{cases} \underset{x_i}{\text{maximize}} \quad \pi_i(x_i|x_{-i},q) = (p(q)-c_i)q_i + (f-p(q))x_i & \text{(1a)} \\ \text{subject to:} \quad q_i \in \arg\underset{q_i}{\text{maximize}}\ \pi_i(q_i|q_{-i},x) = (p(q)-c_i)q_i + (f-p(q))x_i & \text{(1b)} \end{cases}$$

Here q_i is the output of the generating firm in MWh, x_i is the level of hedge cover (MWh). Parameter f is the forward price ($/MWh) and c_i is the marginal cost of the firm ($/MWh). Price is defined by the residual demand curve parameters and the total production output:

$$p(q) = \beta - \alpha Q. \tag{2}$$

Parameters β and α describe the intercept and slope of the residual demand curve respectively and are assumed to be positive and random. Q is the total production in the market. We will focus on the case of firms with market power, that is, firms which face a downward sloping residual demand curve (i.e., $\alpha \geq 0$). In addition we will assume that it is always desirable to produce some output (i.e., $\beta > c$).

3.1. The Hedging Decision of a Single Dominant Firm

We start by focusing on the hedging decisions of a single generating firm operating in a wholesale market with a degree of market power, $Q = q_i$. The problem as given in (1) can be solved by backward induction. The optimal solution to the lower-level optimization problem (1b) is found by taking the first-order conditions with respect to q_i:

$$\frac{\partial \pi_i(q_i|x_i)}{\partial q_i} = -2\alpha q_i + \beta - c_i + \alpha x_i = 0. \tag{3}$$

The corresponding optimal lower-level output parametrized by hedge level x_i is as follows:

$$q_i(x_i) = \frac{1}{2\alpha}(\beta - c_i + \alpha x_i). \tag{4}$$

Combining (2) and (4) we get an expression for price:

$$p(x_i) = \frac{1}{2}(\beta + c_i - \alpha x_i). \tag{5}$$

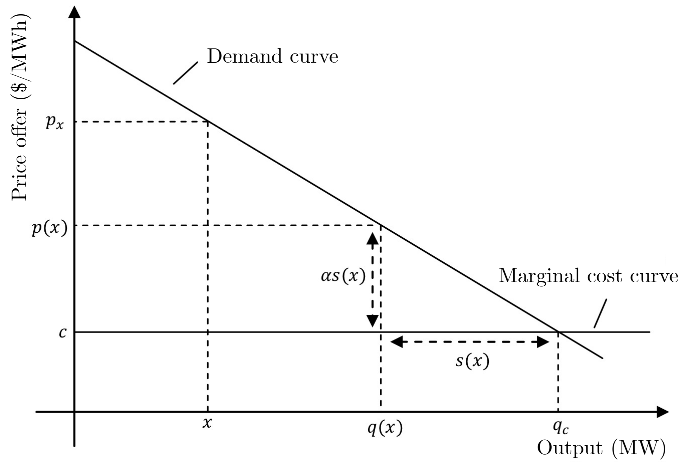


Figure 5 The profit-maximizing price-quantity combination as a function of the hedge level.

We can express this result another way: given linear demand and constant marginal cost, the profit-maximizing quantity lies exactly halfway between the hedge level x_i and the point of intersection of the demand and the marginal cost curves, denoted $q_c = (\beta - c)/\alpha$. The profit-maximizing price lies exactly halfway between the marginal cost c and the price corresponding to the hedge level x_i, denoted $p_x = \beta - \alpha x_i$. This is illustrated in Figure 5, where $s(x)$ expresses the distance between q_c and $q(x)$: $s(x_i) = (\beta - c_i - \alpha x_i)/2\alpha$.

We insert expressions (4) and (5) into the upper-level profit function (1a). The expected profit as a function of the hedge level can then be simplified using the definition of q_c above:

$$E[\pi_i(x_i)] = \frac{1}{4}[\overline{\alpha} x_i^2 + (4f - 2c_i - 2\overline{\beta})x_i + (\overline{\beta}^2 - 2\overline{\beta} c_i + c_i^2)/\overline{\alpha}] \tag{6a}$$

$$= \frac{1}{4}[\overline{\alpha} x_i^2 - 2(\overline{\beta} - c_i)x_i + E[\alpha q_c^2]] + (f - c_i)x_i. \tag{6b}$$

Here $\overline{\alpha} = E[\alpha]$ and $\overline{\beta} = E[\beta]$ are the mean values of α and β. In Sections 3.1.1 and 3.1.2 we distinguish two cases regarding the forward contracts' price f depending whether the hedge level of the firm is observable to traders in the hedge market.

3.1.1. Hedge price independent of firm's hedge level. The hedge level of a firm is typically commercial-in-confidence and not available to traders in the hedge market. Also the traded volumes in the hedge market are typically much higher than those of the spot market. To

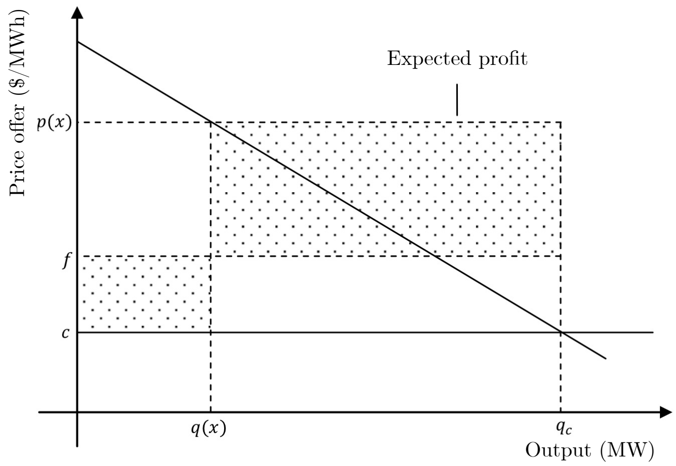


Figure 6 The expected profit of the generating firm as a function of its level of output.

proceed, therefore, we make the assumption that, in the first instance, the price at which hedge contracts can be traded is invariant to small changes in the hedge level. We will look for a rational expectations equilibrium in which trader's forecasts of the hedge level are confirmed ex post.

Since the coefficient $\overline{\alpha}$ is positive, it follows that the expected profit given in (6) is U-shaped in the hedge level and accordingly the choice of hedge level x, which maximizes the expected profit, will be an all-or-nothing decision. The firm will either choose to be hedged to a very high level or will choose to be hedged to a very low (and possibly negative) level.

This can be illustrated graphically in the special case in which the uncertainty in the residual demand function is such that the value q_c is constant. In this case the demand curve pivots around the quantity-price combination (q_c, c_i). The firm faces a trade-off between hedging its output or producing in the spot market. The profit expression can be rewritten as:

$$\pi_i(x_i) = (p(x_i) - f)(q_c - q_i(x_i)) + (f - c_i)q_i(x_i). \tag{7}$$

When the generating firm makes a profit-maximizing choice of output, it receives the hedge price for its output q_i, plus the difference between the spot price and the hedge price for the remaining output up to the level q_c. We know from the discussion above that this profit is maximized at the extremes of the range. This is illustrated in Figure 6.

We suppose that there is some permissible range $[x^{min}, x^{max}]$ for the hedge level where $x^{max} > x^{min}$. The firm will choose the maximum level of hedging if the expected profit in case of maximum hedging is greater than the expected profit in case of minimum hedging:

$$E[\pi_i(x^{max})] - E[\pi_i(x^{min})] \overset{(1a),(2)}{=} (p^{max} - c_i)\frac{\beta - p^{max}}{\alpha} + (f - p^{max})x_i^{max} - (p^{min} - c_i)\frac{\beta - p^{min}}{\alpha}$$

$$- (f - p^{min})x_i^{min} = \frac{1}{\alpha}((p^{max} - p^{min})(p^{max} + p^{min}) + (x_i^{max} - x_i^{min})f\alpha) \overset{(5)}{=}$$
$$= \frac{1}{2}(x_i^{min} - x_i^{max})(p^{max} + p^{min}) + f(x_i^{max} - x_i^{min}). \tag{8}$$

We conclude that the firm will choose the maximum level of hedging if and only if the forward price is greater than the average of the future expected spot price when the hedging is at each end of the range:

$$E[\pi_i(x^{max})] > E[\pi_i(x^{min})] \iff f > \frac{p^{max} + p^{min}}{2}. \tag{9}$$

Here $p^{max} = E[p(x^{max})] = \frac{1}{2}(\overline{\beta} + c - \overline{\alpha}x^{max})$ and $p^{min} = E[p(x^{min})] = \frac{1}{2}(\overline{\beta} + c - \overline{\alpha}x^{min})$ are the expected price outcomes when the firm chooses the maximum and minimum hedging levels, respectively. This is because $x^{max} > x^{min}$ and $p^{max} < p^{min}$. We can formulate the following proposition:

PROPOSITION 1. *Under the assumption of linear demand and constant marginal cost, there is no rational expectations or self-fulfilling equilibrium in pure strategies in the level of hedge cover of the firm.*

Proof: Let us assume that traders in the hedge market expect the firm to choose the minimum level of hedging x^{min}. The corresponding expected price outcome is $E[p(x^{min})] = p^{min}$. Assuming effective competition between traders in the hedge market, the hedge price will be forced to the level $f = E[p(x^{min})] = p^{min}$. But, by condition (9), given this hedge price the profit-maximizing level of hedge cover is x^{max}, contradicting the original assumption. This proves that there cannot be an equilibrium in which traders expect the firm to choose the minimum level of hedging. Similar arguments show that there cannot be an equilibrium in which traders expect the firm to choose the maximum level of hedging. $\square$

Here self-fulfilling equilibrium is a pair of hedge level and hedge price (x_i, f) such that, given the hedge price f, x_i is the profit-maximizing hedge level for the generator and, simultaneously, given the hedge level x_i, the hedge price is equal to the expected future spot price that is $f = E[p(x_i)]$. Assuming risk-neutrality, although there is no equilibrium in pure strategies, there is a mixed-strategy equilibrium. Assume traders expect that the generating firm chooses the minimum level of hedge cover with probability 0.5 and the maximum level of hedge cover with probability 0.5. In this case, the hedge traders will set the hedge price equal to the expected future price outcome: $f = \frac{1}{2}(p^{max} + p^{min})$. Given this hedge price, the generating firm will be indifferent between choosing the maximum or minimum level of hedge cover, which is consistent with the initial hypothesis.

3.1.2. Hedge price dependent on firm's hedge level. Assume that given a level of hedge cover by the dominant firm, arbitrage in the hedge market pushes the hedge price to be equal to the expected future spot price, so that $f = E[p(x)]$. As we have already noted, we do not consider this likely as the hedge position of each generator is a commercial secret and hedge-market traders do not have access to the hedge position of the generators in the market at the time they are making bids and offers. We include this case to highlight the differences in implications and for consistency with the previous research literature.

We assume that the uncertainty in demand has the property that q_c is constant. Furthermore, we assume $q^{min} \le \frac{1}{2}q_c \le q^{max}$ which implies that $x^{min} \le 0 \le x^{max}$. This assumption is necessary to ensure that the profit-maximizing level of hedge cover is an interior maximum. We rewrite expression (6b) using $f = E[p(x_i)]$ and (5):

$$\begin{aligned} E[\pi_i(x)] = &\frac{1}{4}[\overline{\alpha}x_i^2 - 2(\overline{\beta} - c_i)x_i + E[\alpha q_c^2]] + (E[p(x_i)] - c_i)x_i \overset{(5)}{=} \frac{1}{4}[\overline{\alpha}x_i^2 - 2(\overline{\beta} - c_i)x_i \\ &+ E[\alpha q_c^2] + 2(\overline{\beta} + c_i - \overline{\alpha}x_i)x_i - 4c_i x_i] = \frac{\overline{\alpha}}{4}(q_c^2 - x_i^2). \end{aligned} \tag{10a}$$

Under the assumptions above this expression has a unique interior maximum at $x_i = 0$. In other words, a risk-neutral dominant firm has a unique interior profit-maximizing equilibrium choice to be completely unhedged. This result is consistent with the conclusions in Allaz and Vila (1993) and Newbery (2008).

3.2. Hedging in a Two-Stage Oligopoly

In this section we extend the analysis to a Cournot oligopoly. We assume a set of capacity-constrained generators producing the total output K. Further we assume a set of unconstrained generators N. The ith generator ($i \in N$) is assumed to produce the output q_i. The capacity constraints are not binding by definition. The total output in the wholesale spot market is therefore $Q = K + \sum_{i=1}^{n} q_i$. Each generating firm solves optimization problem (1). The first order optimality conditions of the lower-level in case of oligopoly are as following:

$$\frac{\partial \pi_i(q_i|x_i)}{\partial q_i} = -\alpha q_i - \alpha(\sum_{i \in N} q_i + K) + \beta - c + \alpha x_i = 0, \quad \forall i. \tag{11}$$

Solving a system of first order conditions for each generator, we get an expression for profit-maximizing output, parametrized by the hedging decision of generator x_i and other generators x_{-i}:

$$q_i(x_i, x_{-i}) = \frac{1}{\alpha(n+1)}[\beta - \alpha K + (n+1)(\alpha x_i - c_i) - \sum_{i \in N}(\alpha x_i - c_i)], \quad \forall i. \tag{12}$$

In order to simplify the expression, we introduce the following notation:

$$X = \sum_{i \in N} x_i, \quad \overline{c} = \sum_{i \in N} c_i/n, \quad q_{\overline{c}} = (\beta - \overline{c})/\alpha, \tag{13a}$$

$$s(X) = (q_{\overline{c}} - X - K)/(n+1). \tag{13b}$$

We rewrite expression (12) and derive the corresponding price formulation:

$$q_i(X, x_i) = s(X) + x_i + (\overline{c} - c_i)/\alpha, \tag{14a}$$

$$p(X) = \alpha s(X) + \overline{c}. \tag{14b}$$

For tractability, we keep the assumption that $q_{\overline{c}}$ is constant. Generalizing expression (1a), the profit of generator i in the first stage of the game is therefore:

$$\begin{aligned}\pi_i(X, x_i) &= (p(X) - c_i)q_i(X, x_i) + (f - p(X))x_i = (p(X) - c_i)(q_i(X, x_i) - x_i) + (f - c_i)x_i \\ &= \alpha(s(X) + (\overline{c} - c_i)/\alpha)^2 + (f - c_i)x_i. \end{aligned} \tag{15}$$

Since $s(X)$ is linear in x_i, the expected profit is U-shaped in x_i. The profit-maximizing choice of x_i is not in the interior, but at the extremes.

3.2.1. Symmetric Cournot oligopoly. In this section we consider a specific case of a symmetric oligopoly, where $c_i = c_{-i} = c$. We focus on the hedging decision of firm i. The corresponding hedge level of other firms is X_{-i}. There are corresponding minimum and maximum level of hedging for firm i which we denote $x_i^{min}(X_{-i})$ and $x_i^{max}(X_{-i})$. Firm i will choose the maximum level of hedging if and only if the expected profit $E[\pi_i(x_i^{max}, X_{-i})] > E[\pi_i(x_i^{min}, X_{-i})]$. Now, we can write the difference in the expected profit in the two cases as follows:

$$
\begin{aligned}
E[\pi_i(x_i^{max}, X_{-i})] - E[\pi_i(x_i^{min}, X_{-i})] &= \bar{\alpha} s(x_i^{max}, X_{-i})^2 - \bar{\alpha} s(x_i^{min}, X_{-i})^2 + (f-c)(x_i^{max} - \\
x_i^{min}) &\overset{(14a)}{=} s(x_i^{max}, X_{-i})(p^{max} - c) - s(x_i^{min}, X_{-i})(p^{min} - c) + (f-c)(x_i^{max} - x_i^{min}) \\
&\overset{(13b)}{=} s(x_i^{max}, X_{-i})p^{max} - s(x_i^{max}, X_{-i})p^{min} - \frac{np^{min}(x_i^{max} - x_i^{min})}{n+1} - \frac{c(x_i^{max} - x_i^{min})}{n+1} \\
+ f(x_i^{max} - x_i^{min}) &\overset{(14b)}{=} s(x_i^{max}, X_{-i})(\alpha s(x_i^{max}, X_{-i}) - \alpha s(x_i^{min}, X_{-i})) - (x_i^{max} - x_i^{min}) \Bigg(- f \\
+ \frac{(np^{min} + c)}{n+1} \Bigg) &\overset{(13b)}{=} (x_i^{max} - x_i^{min}) \left(f - \frac{n(p^{max} + p^{min}) - (n-1)c}{n+1} \right).
\end{aligned}
\tag{16}
$$

Generalizing (16), the firm will choose the maximum level of hedging if and only if:

$$
f - c > \frac{n(p^{max} - c + p^{min} - c)}{n+1}. \tag{17}
$$

Further we again distinguish two cases, when the hedge price depends on firms' hedge level and when it does not.

3.2.2. Symmetric oligopoly: Hedge price dependent on firms' hedge level. We look for a rational expectations equilibrium in pure strategies. We can prove the following proposition:

PROPOSITION 2. *Under the assumption of linear residual demand, constant marginal cost and uncertain demand with the property that the profit-maximizing level of output of each firm is independent of the realization of the demand parameters, given a set of n identical generators, and assuming that firms play a Cournot game in quantities in the spot market and play a Cournot* *game in hedge levels in the hedge market, then there is no rational expectations equilibrium in pure strategies in the hedge market unless the expected future wholesale spot price when all generators produce at their maximum output exceeds the marginal cost by the following margin:*

$$p - c > \frac{n}{n+1}[2(\beta - \alpha K - c) - \alpha n(q^{max} + q^{min})].$$

Proof: Let us suppose the hedge traders assume that all generators will hedge to the maximum level. The resulting hedge price will be $f = p^{max}$. Using (17), we see that choosing the maximum level of hedge contracting is a Nash equilibrium if and only if:

$$\begin{aligned} & f - c > \frac{n(p^{max} - c + p^{min} - c)}{n+1} \\ & \Longleftrightarrow p^{max} - c > \frac{n}{n+1}[2(\beta - \alpha K - c) - \alpha n(q^{max} + q^{min})] \end{aligned}$$

The terms $(\beta - \alpha K - c)$ and $(q^{max} + q^{min})$ are positive by the previous assumptions. This means that depending on the setup of the parameters the inequality may be satisfied, when p^{max} exceeds c by a sufficient margin. The same argument holds for the case $f = p^{min}$. □

3.2.3. Symmetric oligopoly: Hedge price dependent on firms' hedge level. We now consider the final case in which the traders are assumed to be able to observe the hedging level of all the generators with market power, so that the hedge price can perfectly track the expected future spot price. We can write the expected profit as follows:

$$\begin{aligned} \pi_i(x_i|X_{-i}) &= (p(X) - c_i)q_i(X, x_i) + (f - p(X))x_i \\ &\overset{(14)}{=} \alpha\left(s(X) + \frac{\bar{c} - c_i}{\alpha}\right)\left(s(X) + x_i + \frac{\bar{c} - c_i}{\alpha}\right). \end{aligned} \tag{18}$$

PROPOSITION 3. *Suppose we have a Cournot oligopoly of n capacity-unconstrained firms with possibly varying marginal costs and capacities, facing a capacity-constrained fringe of firms and a linear demand, with the property that the volume of demand at the average marginal cost is constant. Each firm will choose to be hedged the same proportion of a weighted-average of its output: $1 - 1/n$.*

Proof: The expected profit expression (18) has a unique interior maximum since $s(X)$ is a linear function of x_i with a coefficient between -1 and zero. The first order condition can be formulated as follows:

$$\frac{\partial \pi_i(x_i|X_{-i})}{\partial x_i} = \frac{-2\alpha s(X) - 2(\bar{c} - c) - \alpha x_i}{n+1} + \alpha s(X) + (\bar{c} - c) = 0. \tag{19}$$

From the first order condition for x_i we find that:

$$x_i(X) = (n-1)\left(s(X) + \frac{\bar{c} - c_i}{\bar{\alpha}}\right).$$

Using expression (14a) we have that:

$$q_i(X) = s(X) + \frac{\bar{c} - c_i}{\bar{\alpha}} + x_i(X) = n\left(s(X) + \frac{\bar{c} - c_i}{\bar{\alpha}}\right)$$

Hence it follows that in the Nash equilibrium, each firm chooses to be hedged a fixed proportion of a weighted average of its output, with the proportion increasing as the number of firms in the market increases (consistent with Newbery (2008)).

$$x_i(X) = \left(1 - \frac{1}{n}\right) q_i(X).$$

□

3.3. Quadratic cost functions

The derivations in Sections 3.1 and 3.2 have assumed linear cost functions and, therefore, constant marginal costs. However, it is commonly recognized that the cost function in electricity markets is a steep polynomial function, sometimes approximated as quadratic. In this section we extend the previous results for the case of quadratic cost function $C(q_i) = a_i q_i^2 + c_i q_i + b_i$. A decision-making problem of profit-maximizing generating company with quadratic costs can be formulated as follows:

$$\forall i: \begin{cases} \underset{x_i}{\text{maximize}} \quad \pi_i(x_i|x_{-i}, q) = p(q)q_i - a_i q_i^2 - c_i q_i - b_i + (f - p(q))x_i & \text{(20a)} \\ \text{subject to:} \quad q_i \in \arg \underset{q_i}{\text{maximize}}\, \pi_i(q_i|q_{-i}, x) = p(q)q_i - a_i q_i^2 - c_i q_i & \\ \qquad\qquad -b_i + (f - p(q))x_i & \text{(20b)} \end{cases}$$

3.3.1. Single firm. Assuming quadratic cost function the optimal lower-level output and the corresponding price, parametrized by hedge level x_i are as follows:

$$q_i(x_i) = \frac{1}{2(\alpha + a_i)}(\beta - c_i + \alpha x_i), \tag{21a}$$

$$p(x_i) = \frac{1}{2(\alpha + a_i)}(\alpha(\beta + c_i - \alpha x_i) + 2a_i\beta). \tag{21b}$$

In the following corollaries we show that the results obtained for the case with linear costs hold in the case of quadratic costs.

Corollary 1. ***Independent hedge price for single dominant firm with quadratic costs.*** *A single dominant generating firm with quadratic cost function has no pure equilibrium hedging strategy.*

Proof: The firm will choose to hedge to the maximum level if $E[\pi_i(x^{max})] > E[\pi(x^{min})]$. In Appendix A, expression (26) is simplified to (8). Therefore, Proposition 1 holds for the case of quadratic costs and there is no equilibrium in pure strategies. □

Corollary 2. ***Hedge price equals the expected spot price for single dominant firm with quadratic costs.*** *A single dominant firm with quadratic cost function and hedge price exactly equal to the expected spot price has a unique optimal strategy to choose a zero hedge cover.*

Proof: The profit expression for the case when hedge price is equal to the spot price is demonstrated in expression (27). It has a unique maximum at $x_i = 0$, which agrees with the previous result for linear cost function. □

3.3.2. Oligopoly. For completeness we consider the case of quadratic cost for the oligopolistic situation. There is a set of capacity-constrained generators producing the total output K and N unconstrained generators. The first order optimality conditions of the lower-level optimization problem in case of oligopoly are as following:

$$\frac{\partial \pi_i(q_i|x_i)}{\partial q_i} = -\alpha q_i - \alpha(\sum_{i \in N} q_i + K) + \beta - b_i - 2a_i q_i + \alpha x_i = 0, \quad \forall i. \tag{22}$$

First we consider the case, when hedge cover is not observable to the traders and therefore the hedge price f does not depend on the hedge level x_i. Solving a system of these expressions for all firms and deriving the optimal profit expression provides us with the following proposition:

PROPOSITION 4. ***Independent hedge price in asymmetric oligopoly with quadratic costs.*** *In asymmetric oligopoly with N unconstrained generating firms and unobservable hedge cover, there may exist an equilibrium at which the firms take an all-or-nothing hedging decision.*

Proof: The oligopoly case with quadratic costs presents a more complex case than the one with linear cost functions. We show that the expression corresponding to the coefficient of the quadratic hedging term in the profit formulation of any strategic unconstrained firm i may become negative for sufficiently large coefficients of quadratic cost functions a_i and a high number of generating firms. This means that depending on the value of parameters and the number of competing frms, the equilibrium strategy may be an all-or-nothing solution. The derivation is provided in Appendix B.1. □

We also consider the case of perfectly observable hedge cover. In this case the forward price f will be exactly equal to the expected market price p. We obtain the following proposition:

PROPOSITION 5. ***Hedge price equals the expected spot price in asymmetric oligopoly with quadratic costs.*** *In asymmetric oligopoly with N unconstrained generating firms and observable hedge cover there may be an equilibrium at which the generating firms will prefer an all-or-nothing solution.*

Proof: We show that profit function is quadratic. The coefficients corresponding to the quadratic terms may or may not be positive, depending on the value of parameters. We conclude that there might exist an equilibrium at which generating firms may choose an all-or-nothing hedging decision. The derivation is provided in Appendix B.2. □

4. Numerical results

In this section we illustrate the closed-form results obtained in Section 3. We also study more realistic market situations including an asymmetric oligopoly and an electricity market with various levels of competition between generators.

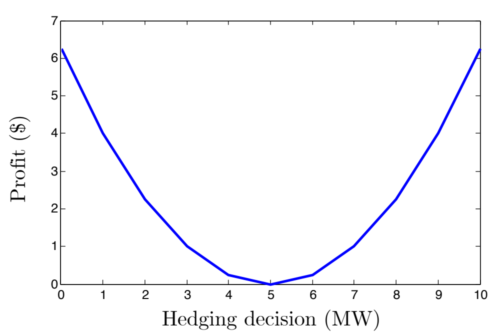
4.1. Hedging decision of a single dominant generating company

We have seen in the previous section that profit-maximizing company faces an all-or-nothing hedging decision. Assume a company with marginal costs $c = 5$ \$/MWh, facing a decision to hedge with the forward price $f = 5$ \$/MWh. The demand is characterized by elasticity $\alpha = 1$ \$/MWh^2 and demand intercept $\beta = 10$ \$/MWh.

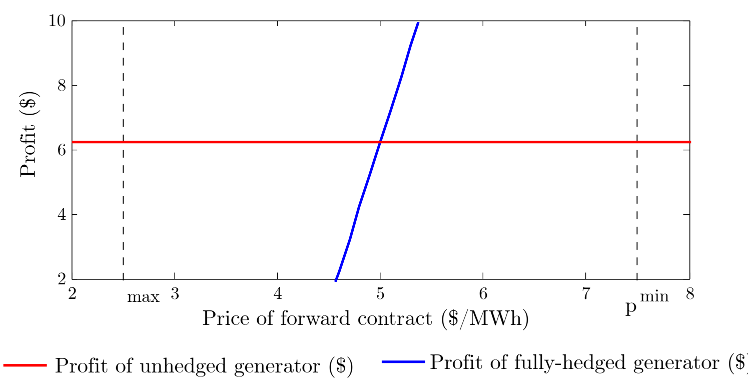
The profit of generator with respect to the hedging decision is presented in Figure 7(a). Assuming that there is a range of available hedging decisions $[x^{min}, x^{max}]$, the generator is facing a symmetric all-or-nothing decision. The values maximizing the profit lie on the borders of the available range of forward contracts $[x^{min}, x^{max}]$.

In Figure 7(b) we plot the profit of the generator as it chooses all-or-nothing hedging decisions for different prices of forward contracts. We can directly observe that the all-hedging decision becomes more profitable, when the price of forward contracts exceeds the average of the prices in case of minimum and maximum hedging (p^{min} and p^{max}) i.e. the point in the middle in the graph. We can also conclude that the profit for no-hedging decisions remains constant in forward price f as it does not depend on it.

For completeness of the discussion in Figure 8 we plot the profit of the same dominant company for the situation when hedge cover is publicly-available information. In that case the price of the



(a)



(b)

Figure 7 The expected profit of the generating firm as a function of its hedging decision: hedge cover is confidential information.

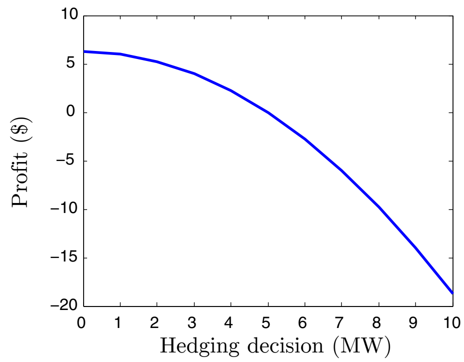


Figure 8 The expected profit of the generating firm as a function of its hedging decision: hedge cover is public information.

forward contract is set equal to the expected spot price. We see that in contrast with Figure 7 there is only one equilibrium, corresponding to the case when the hedge cover is zero.

4.2. Hedging decision in a symmetric oligopoly

Now we look at a case of symmetric oligopoly with two companies – $u1$ and $u2$ – with marginal costs $c = 5$ $/MWh, facing the same demand function as in Section 4.1. Using simulations we obtain the price in case of maximum hedging $p^{max} = 0$ $/MWh and in case of minimum hedging $p^{min} = 6.67$ $/MWh. According to the result in (17), the firms will choose to hedge to the maximum level if $f > 2.78$ $/MWh[3].

The results in Figure 9 present the reaction functions of two competitors. The intersection of the lines represent equilibria. As expected, on the leftmost plot the only equilibrium is when all

[3] $f - 5 > (2(0 - 5 + 6.67 - 5))/(2 + 1) \leftrightarrow f > 2.78$

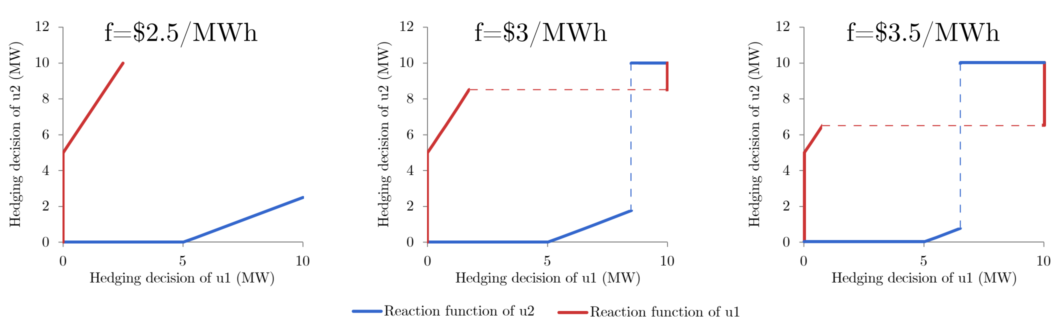


Figure 9 Reaction functions of symmetric firms.

generators are unhedged. However, when the price of forward contracts exceeds 2.78 \$/MWh, there appear more equilibria. The only pareto optimal equilibrium (the one that maximizes the profits of all generators) is to enter the market fully hedged.

4.3. Hedging decision in an asymmetric oligopoly

It is often useful from a regulatory viewpoint to evaluate the desired levels of hedging in an asymmetric oligopoly, or an oligopoly where one of the competitors has lower marginal costs. The closed-form solution is difficult in interpretation, as many possible cases have to be analyzed (Ke 2011). Therefore, in this section we discuss some intuitions for practical use. For this section we assume a formulation equivalent to the one discussed in Section 4.2, except that the costs of the competitors are different: $c_1 = 2$ \$/MWh, $c_2 = 5$ \$/MWh.

The results presented in Figure 10 demonstrate that the general conclusions for a symmetric oligopoly also hold for the asymmetric oligopoly as specified in our example. For a high-enough price of forward contracts there is an equilibrium at which the companies may choose to hedge to a maximum level. Another interesting observation is that the firm with lower marginal costs is less prone to use hedging, compared to the more expensive firm. This can be explained by the competitive advantage of the cheaper producer (it prefers to go unhedged to the spot market, as it has more market power). This agrees with the conclusions in Sánchez et al. (2009), where the authors use an agent-based learning model to show that larger companies (the companies with

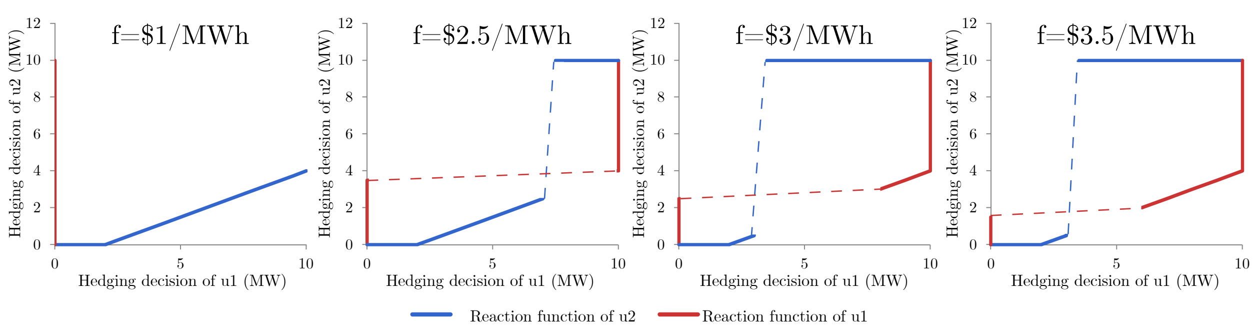


Figure 10 Reaction functions of asymmetric firms.

competitive advantage) choose to go unhedged to the spot market in order to exercise market power, while smaller companies tend to have hedge contracts.

4.4. Various levels of competition: conjectured price response parameter

The range of competition intensities can be captured by conjectural variations, as first introduced in Bowley (1924). Conjectural variation reflects the belief of a firm regarding the response of the competitors to the variations in firm's output or price. The approach allows a range of strategic outcomes from competitive to cooperative.

We define the conjectural variation parameters as $\Phi_{j,i}$. These represent firm i's belief about how another firm j changes its production in response to a change in i's production. Therefore:

$$\Phi_{j,i} = \frac{dq_j}{dq_i}, \quad i \neq j \tag{23a}$$

$$\Phi_{i,i} = 1. \tag{23b}$$

Using expressions (23a)-(23b) in a price function, where price is a function of supply in the market $p(q_i, q_{-i})$, such as the function given in (2), we obtain:

$$\frac{dp(q_i, q_{-i})}{dq_i} = -\alpha \sum_{i,j} \Phi_{j,i} = -\alpha(1 + \sum_{j \neq i} \Phi_{j,i}). \tag{24}$$

In our application we consider a global conjectural variation Φ which represents the combined reaction of all competitors: $\Phi = \sum_{j \neq i} \Phi_{j,i}$.

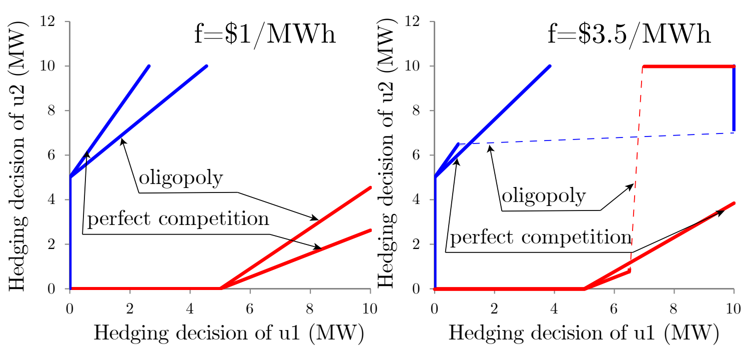


Figure 11 Reaction functions of symmetric firms under different competition intensities θ.

We can now define a conjectured price response parameter θ_i as the company i's belief concerning its influence on price $p(q_i, q_{-i})$. We express the conjectured price response as:

$$\theta_i = \alpha(1+\Phi) = -\frac{dp(q_i, q_{-i})}{dq_i}. \tag{25}$$

Different levels of the conjectured price response parameter correspond to different market competition intensities. We assume θ_i varying from 0, which corresponds to perfect competition in the market to α, which represents the Cournot oligopoly (Daxhelet 2008). This range reflects the most common electricity market structures. Further, we assume symmetric conjectured price responses $\theta = \theta_1 = \theta_2$.

Figure 11 shows the reaction functions of symmetric firms under different competition intensities, represented by conjectural variations. When the forward price is low the companies prefer to enter the spot market completely unhedged, both under perfect and oligopolistic competition. However, as the price of forward contract increases the companies in an oligopoly face all-or-nothing decision.

5. Future work: Hedging decision of a single firm with wind power uncertainty

In Sections 3 and 4 we have looked at a case of symmetric probability distribution function of demand parameters, when the exact realization is not affecting the optimal hedging decision. However, power systems are increasingly being affected by the integration of renewable energy sources, wind power in particular, with a substantial part being active on the demand-side as embedded generation. The share of demand covered by wind energy in European Union has reached 10.4% in 2016 (Nghiem and Mbistrova 2017). Since demand has a direct relationship with market price (Coulon et al. 2013), it is important to account for the statistical characteristics of wind power, when devising an optimal hedging strategy. It is also suggested in Bunn and Chen (2013) that wind power brings radical market structure changes and, therefore, has to be accounted for in the hedging decision analysis.

In this section we indicate the direction for extending the work proposed in this paper to the case of wind-integrated power systems. The authors in Bessembinder and Lemmon (2002) demonstrate how the optimal hedging decision of a generating firm depends on forecast output and on the skewness of power demand. The authors consider high order polynomial cost functions. This implies that marginal production costs increase very fast in demand, reflecting the fact that industry employs an array of production technologies and fuel sources from hydro to natural gas. In this section we consider the skewness of the demand, originating from the wind power integration.

We assume that the electricity market has high share of wind generation with the mean value covering 20% of the demand. Wind power generation can be represented by beta distribution (Morales et al. 2013) with mean value $\beta^{wind} = 2.5$ \$/MWh subtracted from the net demand (Biggar and Hesamzadeh 2014). The profit calculation is presented in Figure 12. System demand is partially covered by wind generation, having a beta distribution. Accordingly, net demand also has beta-distribution as shown in the top plot in Figure 12. This makes the profit-maximizing hedging decision of generator asymmetric. In the bottom plot in Figure 12 we observe that with high wind integration, the generator obtains higher profits by going to the spot market without hedge cover. This effect can be attributed to the shape of the beta distribution function setting higher probability to lower production outcomes of wind generation.

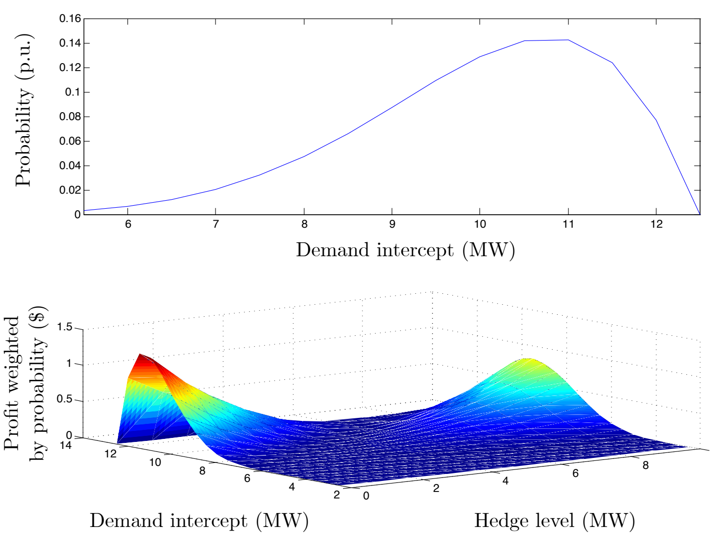


Figure 12 The expected profit of the generating firm as a function of its hedging decision and demand intercept (demand intercept has beta distribution).

6. Conclusion

Deciding upon the appropriate level of hedging is a crucial operational decision in supply chain management. The existing research on the equilibrium level of hedging in an electricity market with risk neutral players has assumed that hedge prices are equal to expected future spot prices. We argue that this requires detailed knowledge of the hedge position of dominant firms, and that this does not generally hold in the forward markets. This transparency of information on contract positions is unlikely to be available to hedge traders. As a consequence, we argue that it is more appropriate and important to assume that hedge prices are independent of hedging decisions. We show that under this assumption, in a market dominated by a firm with a high degree of market power there may be *no* equilibrium hedge position in pure strategies. Instead, the hedging decision of a dominant generator will be all-or-nothing, depending on the price at which the dominant generator can sell its hedge contracts. We therefore suggest that electricity wholesale markets featuring a dominant generator are prone to a lack of liquidity in the hedge market and potentially volatile wholesale spot and forward prices. This intuition reflects the pattern of outcomes observed in the South Australian region of the Australian National Electricity Market.

The model also shows that as the number of firms in the market increases, there arises an equilibrium in which all the firms choose to be fully hedged. Our resutls hold when we consider the cases of linear and quadratic cost functions. We study the more general case of asymmetric oligopolies numerically. We arrive at similar conclusions as for the symmetric case (i.e. the generators may choose to prefer hedging, if the price of the forward contract is sufficiently high). An extension includes the effect of market structure on competition, which we model using conjectured price response. A further extension, indicated as a future work, includes the specific characteristics of uncertainty, e.g. wind power, on hedging decision of the generator. Our preliminary results show that the skewness in the probability distribution function of wind power production makes the hedging decision of generators tend towards zero hedge cover.

In this research we assumed the risk neutrality of market participants. We have looked at hedging from a strategic perspective, and not as a consequence of risk aversion. Evidently, in many corporate contexts, risk management requirements will direct adequate hedge cover according to Value-at-Risk or other compliance metrics. Also, the need to report adequate hedging to investors at periodic intervals is often an important part of managing investor relations in large corporations. All of these risk-induced motivations for hedging will moderate the conclusions of our research. Nevertheless, the analysis presented here involves a new consideration of the effect of a lack of transparency in the analysis of strategic forward trading, which would appear to be very relevant in practice. It provides new insights on how particular companies with market power may operate their supply chains with and without hedges and what this may mean for market prices. In general, it raises a policy and regulatory question about greater transparency in forward markets and how that might be achieved.

Appendices

A. Supplementary mathematical expressions for Section 3: Single dominant company

A.1. Independent hedge price

A single dominant firm with quadratic cost function will choose to hedge to maximum level, if $E[\pi_i(x^{max})] > E[\pi_i(x^{min})]$, or:

$$\begin{aligned}
&E[\pi_i(x^{max})] - E[\pi_i(x^{min})] = (p^{max} - c_i)\frac{\beta - p^{max}}{\alpha} - a_i(\frac{\beta - p^{max}}{\alpha})^2 + (f - p^{max})x_i^{max} - (p^{min} - \\
&c_i)\frac{\beta - p^{min}}{\alpha} + a_i(\frac{\beta - p^{min}}{\alpha})^2 - (f - p^{min})x_i^{min} = \frac{1}{\alpha}(p^{max}\beta - (p^{max})^2 - c_i\beta + c_i p^{max} - p^{min}\beta \\
&+ (p^{min})^2 + c_i\beta - c_i p^{min}) + f x_i^{max} - p^{max}x_i^{max} - f x_i^{min} + p^{min}x_i^{min} + a_i((\frac{\beta - p^{min}}{\alpha})^2 - \\
&(\frac{\beta - p^{max}}{\alpha})^2) = \frac{1}{\alpha}(p^{max}(\beta - p^{max} - \alpha x_i^{max} + c_i) - p^{min}(\beta - p^{min} - \alpha x_i^{min} + c_i)) + \\
&f(x^{max} - x^{min}) + \frac{a_i}{\alpha^2}(\beta^2 - 2\beta p^{min} + (p^{min})^2 - \beta^2 + 2\beta p^{max} + (p^{max})^2) = \frac{1}{\alpha^2}(p^{max}(\alpha p^{max} \\
&+ a_i p^{max}) - p^{min}(\alpha p^{min} + a_i p^{min})) + f(x_i^{max} - x_i^{min}) = \frac{\alpha + a_i}{\alpha^2}(p^{max} - p^{min})(p^{max} + p^{min}) + \\
&f(x_i^{max} - x_i^{min}) = \frac{1}{2}(x_i^{max} - x_i^{min})(p^{max} + p^{min}) + f(x_i^{max} - x_i^{min}). \qquad (26)
\end{aligned}$$

The final expression is equivalent to the result of (8), therefore Proposition 1 holds for the case of a single dominant firm and quadratic costs.

A.2. Hedge price equals the expected spot price

In the case the hedge price is exactly equal to the expected spot price, profit can be expressed from (20a) using (21a) and (21b):

$$\begin{aligned} E[\pi_i(x)] =& p(x)q_i(x) - a_i q_i^2 - c_i q_i - b_i = \frac{1}{4(a_i+\alpha)}(-\alpha^2 x_i^2 - 4b_i\alpha + b_i^2 - 2c_i\beta + \beta^2 - 4a_i b_i) = \\ & \frac{1}{4(a_i+\alpha)}(-\alpha^2 x_i^2 - 4b_i(a_i+\alpha) + (c_i-\beta)^2). \end{aligned} \tag{27}$$

The unique maximum of this expression is, when $x_i = 0$.

B. Supplementary mathematical expressions for Section 3: Oligopoly

B.1. Independent hedge price

Expression (22) can be rewritten in order to express the production level parametrized by the hedging decision of other generators:

$$\begin{aligned} & \alpha q_i + 2a_i q_i + \alpha \sum_{i\in N} q_i = \beta - \alpha K - c_i + \alpha x_i, \\ & q_i + \frac{\alpha}{2a_i+\alpha}\sum_{i\in N} q_i = \frac{1}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i), \\ & \sum_{i\in N} q_i + \sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\sum_{i\in N} q_i\right) = \sum_{i\in N}\left(\frac{1}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i)\right), \\ & \sum_{i\in N} q_i = \frac{\sum_{i\in N}\left(\frac{1}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)} \\ & q_i = \frac{1}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i) - \frac{\alpha}{2a_i+\alpha}\frac{\sum_{i\in N}\left(\frac{1}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)}, \\ & q_i = \frac{1}{2a_i+\alpha}\left(\beta - \alpha K - c_i + \alpha x_i - \frac{\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i)\right)}{1+\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)}\right). \end{aligned} \tag{28}$$

The corresponding price is:

$$p = \beta - \alpha(\sum_{i\in N} q_i + K) = \beta - \alpha K - \frac{\sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}(\beta - \alpha K - c_i + \alpha x_i)\right)}{1 + \sum_{i\in N}\left(\frac{\alpha}{2a_i+\alpha}\right)}. \tag{29}$$

Combining expressions (28) and (29) we can also write that:

$$q_i = \frac{1}{2a_i+\alpha}(p - c_i + \alpha x_i). \tag{30}$$

Using expressions (29) and (30) in the profit formulation (20a) we find out that the profit expression is a quadratic function of the hedging decision x_i:

$$\begin{aligned}\pi_i =& (p-c_i)q_i - a_i q_i^2 - b_i + f x_i - p x_i = \\ & \frac{p-c_i}{2a_i+\alpha}(p-c_i+\alpha x_i) - \frac{a_i}{(2a_i+\alpha)^2}(p-c_i+\alpha x_i)^2 - b_i + f x_i - p x_i = \\ & \frac{a_i+\alpha}{(2a_i+\alpha)^2}(p-c_i)^2 + \frac{\alpha}{(2a_i+\alpha)^2}(p-c_i)\alpha x_i - \frac{a_i}{(2a_i+\alpha)^2}(\alpha x_i)^2 - b_i + f x_i - p x_i.\end{aligned} \tag{31}$$

Some of the coefficients corresponding to the quadratic terms are positive while the sign of some other coefficients is indeterminate.

This means that two situations are possible: (1) profit function is ∪-shaped and optimal solutions lie on the extremes of the feasible range and (2) profit function is ∩-shaped, which means that there is a single solution, which might not be all-or-nothing decision. Numerical results shown in Figure 13 confirm that there exist cases, when the coefficient corresponding to the quadratic term in (31) is positive, meaning that market may become illiquid in certain situations.

B.2. Hedge price equals the expected spot price

If hedge positions of all generating companies is perfectly observable we can expect that $E[p] = f$. Therefore, the profit expression can be rewritten:

$$\begin{aligned}\pi_i =& (p-c_i)q_i - a_i q_i^2 - b_i = \frac{p-c_i}{2a_i+\alpha}(p-c_i+\alpha x_i) - \frac{a_i}{(2a_i+\alpha)^2}(p-c_i+\alpha x_i)^2 - b_i = \\ &= \frac{1}{2a_i+\alpha}(p-c_i)^2 + \frac{1}{2a_i+\alpha}(p-c_i)\alpha x_i - \frac{a_i}{(2a_i+\alpha)^2}((p-c_i)^2 + 2(p-c_i)(\alpha x_i) + \\ & (\alpha x_i)^2) - b_i = \frac{a_i+\alpha}{(2a_i+\alpha)^2}(p-c_i)^2 + \frac{\alpha}{(2a_i+\alpha)^2}(p-c_i)(\alpha x_i) - \frac{a_i}{(2a_i+\alpha)^2}(\alpha x_i)^2 - b_i.\end{aligned}$$

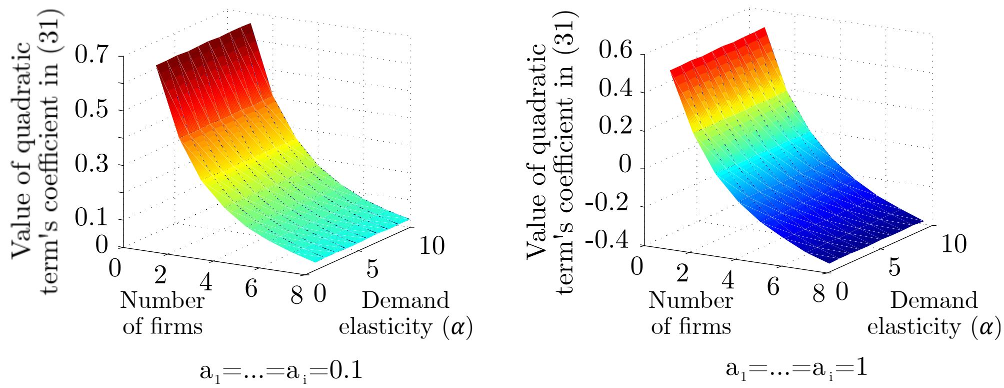


Figure 13 The value of the quadratic term's coefficient in the profit formulation of a firm in the symmetric oligopoly case (31).

As before the profit function is quadratic in the hedging decision. We again encounter the situation, where there might be all-or-nothing solution, depending on the value of the coefficients.

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