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Comparing auction designs where suppliers have uncertain costs and uncertain pivotal status

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We analyze how market design influences bidding in multiunit procurement auctions where suppliers have asymmetric information about production costs. Our analysis is particularly relevant to wholesale electricity markets, because it accounts for the risk that a supplier is pivotal; market demand is larger than the total production capacity of its competitors. With constant marginal costs, expected welfare improves if the auctioneer restricts offers to be flat. We identify circumstances where the competitiveness of market outcomes improves with increased market transparency. We also find that, for buyers, uniform pricing is preferable to discriminatory pricing when producers' private signals are affiliated.

1. Introduction

■ Multiunit auctions are used to trade commodities, securities, emission permits, and other divisible goods. This article focuses on electricity markets, where producers submit offers before the level of demand and amount of available production capacity are fully known. Due to demand shocks, unexpected outages, transmission-constraints, and intermittent output from renewable energy sources, it often arises that an electricity producer is pivotal, that is, that realized demand is larger than the realized total production capacity of its competitors. A producer that is certain to be pivotal possess a substantial ability to exercise market power because it can withhold output

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and push the price up to the reservation price of the consumers. We are interested in how such markets perform and how they are influenced by the auction design. Most electricity markets use uniform pricing, where the highest accepted offer sets the transaction price for all accepted production. A few markets, such as the British real-time market, use discriminatory pricing, where each accepted offer is instead paid its own offer price.[1] There have been calls to switch from uniform to discriminatory pricing in a number of electricity markets (Kahn et al., 2001).

Our model accounts for asymmetric information in suppliers' production costs. Our analysis is, for example, of relevance for European wholesale electricity markets, where the European Commission has introduced regulations that increase the market transparency, so that uncertainties and information asymmetries are reduced.[2] Long before delivery, in forward markets, the uncertainty about future fuel prices is to a large extent a common uncertainty among producers. The relative size of this common uncertainty typically decreases closer to the delivery. In the electricity spot market, an owner of a thermal plant has private information about the actual price paid for its input fuel. Daily natural gas prices can have large uncertainties due to local congestion and local storage constraints in gas pipelines. These circumstances can be even more severe in regions with significant amounts of intermittent wind and solar generation capacity, because natural gas units must make up for any renewable energy shortfall relative to system demand. Moreover, the owner of a thermal plant has private information about the efficiency of its plant, which depends on the ambient temperature, and how the plant is maintained and operated.

The cost uncertainty and the information asymmetry between firms can also be significant in hydro-dominated markets. The opportunity cost of using water stored in the reservoir behind a specific generation unit is normally estimated by solving a stochastic dynamic program based on estimates of the probability distribution of future water inflows and future offer prices of thermal generation units. These firm-specific estimated opportunity costs typically have a significant common component across suppliers. The stochastic simulations also leaves significant scope for differences across market participants in their estimates of the generation unit-specific opportunity cost of water. The uncertainty in this opportunity cost is exacerbated by the possibility of regulatory intervention, especially during extreme system conditions, and each producer's subjective beliefs about the probability of these events occurring during the planning period.

Our model of an electricity market with asymmetric information about supplier costs assumes a multiunit auction with two capacity-constrained producers facing an inelastic demand. Each producer also has an uncertain amount available of generation capacity that is realized after offers are submitted. Demand is also uncertain and realized after offers have been submitted. Both of these sources of uncertainty and when they are realized is consistent with how spot electricity markets operate. Similar to the electricity market model by von der Fehr and Harbord (1993), each firm has a flat marginal cost (independent of output) up to the capacity constraint and must make a flat offer. This is also similar to the Colombian electricity market, where each supplier chooses one offer price for the entire capacity of each generation unit (Wolak, 2009). We generalize von der Fehr and Harbord (1993) by introducing uncertain interdependent costs. Analogous to Milgrom and Weber's (1982) auction for single objects as well as Ausubel et al.'s (2014) and Vives' (2011) models of multiunit auctions, each firm makes its own estimate of production costs based on the private information that it receives, and then submits an offer.[3] As is customary in game theory, we refer to this private information as a private signal. We solve for a unique

[1] In addition, some special auctions in the electricity market, such as counter-trading in the balancing market and/or the procurement of power reserves, sometimes use discriminatory pricing (Holmberg and Lazarczyk, 2015; Anderson, Holmberg, and Philpott, 2013).The US Treasury is an important exception, but otherwise most treasury auctions around the world use discriminatory pricing (Bartolini and Cottarelli, 1997; Brenner, Galai, and Sade, 2009).

[2] According to EU No. 543/2013, the hourly production in every single plant should be published. EU No. 1227/2011 (REMIT) mandates all electricity market participants to disclose insider information, such as the scheduled availability of plants.

[3] Milgrom and Weber (1982) and Ausubel et al. (2014) analyze sales auctions, so in their settings, each agent estimates the value of the good that the auctioneer is selling.

Bayesian Nash Equilibrium (BNE) when signals are drawn from a bivariate distribution that is known to the suppliers.

In our setting with flat marginal costs, inelastic demand, and *ex ante* symmetric producers, the bid constraint that offers must also be flat improves expected welfare. A comparison of our results to Vives (2011) suggests that the bid constraint is particularly beneficial for uniform-price auctions where producers have large common uncertainties in their costs. This is mainly relevant for uniform-price auctions of forward contracts and hydro-dominated electricity markets, where the opportunity cost has a significant common uncertainty and is approximately flat for a wide range of outputs.[4]

For an auction with our bid constraint, we show that the auctioneer would prefer uniform to discriminatory pricing if signals of producers are affiliated. This is related to Milgrom and Weber's (1982) ranking of first- and second-price single-object auctions. In the special case where signals are independent, we find that the two auction formats are revenue-equivalent. This is a generalization of revenue-equivalence results for single-object auctions by Myerson (1981) and Riley and Samuelson (1981). Our results also generalize Fabra, von der Fehr, and Harbord (2006), who prove revenue equivalence for our setting when costs are common knowledge.

Equilibrium offers in a discriminatory auction are determined by the expected sales of the highest and lowest bidder, respectively. A smaller difference between these sales means that both producers are pivotal by a larger margin and equilibrium offers increase. Under our modelling assumptions, the variance in sales after offers have been submitted—due to demand shocks, outages and intermittent renewable production—will not influence the bidding behavior of producers or their expected payoffs in the discriminatory auction. Results are similar for uniform-price auctions, but equilibrium offers in that auction are more sensitive to the variance in sales. The variance in sales due to demand shocks and outages does not influence the ranking of auctions.

We extend our basic model to consider the case that the auctioneer has private cost-relevant information that it can disclose to the two suppliers. For a discriminatory auction, we can show that the auctioneer would benefit from disclosing its information, if its signal and the producers' signals are all affiliated. Intuitively, this should also hold for uniform-price auctions, but in this case, we are only able to prove this when the signals of producers are independent. This is related to the publicity effect that was proven by Milgrom and Weber (1982) for single-object auctions. Vives (2011) finds that markups decrease when nonpivotal producers receive less noisy cost information before competing in a uniform-price auction. It is known from Perry and Reny (1999) that the publicity effect may not hold in multiunit auctions. Still, taken together, these results suggest that publicly available information of relevance for production costs—such as fuel prices, prices of emission permits, and water levels in reservoirs—is likely to improve the competitiveness of market outcomes in electricity markets. Similarly, disclosing detailed historical bid data and/or detailed production data are likely to make production costs more transparent.[5] In addition, information provision about outcomes from financial markets just ahead of the operation of related physical markets should lower the market uncertainty. Similarly, trading of long-term contracts, which help producers predict future electricity prices, should reduce the extent of informational asymmetries among suppliers about the opportunity cost of water.

Extending this logic further, our results suggest that regulatory risks that increase information asymmetries among players about the opportunity cost of water are particularly harmful for competition in hydro-dominated wholesale electricity markets, especially when water is scarce. Thus, we recommend clearly defined contingency plans for intervention by the regulator in case of

[4] Analogously, bidders' marginal valuation of securities is fairly insensitive to the purchased volume and often have a large common value component. This indicates that bid constraints have the potential to increase welfare and auction sales revenues in uniform-price security auctions.

[5] Note that disclosure of individual offers would give detailed information on all plants, not only the marginal plant, for every auction outcome.

extreme system conditions. This could potentially mitigate the extraordinarily high-priced periods that typically accompany low-water conditions in hydro-dominated markets, as has occurred in California, Colombia, and New Zealand.

Because increased transparency reduces the payoff of producers in our model, we would not expect producers to agree to voluntarily disclose production cost-relevant information. This has similarities to Gal-Or (1986) who shows that producers that play a Bertrand equilibrium would try to conceal their private costs from each other. Moreover, increased transparency would only be helpful up to a point, because there is a lower bound on equilibrium markups when a producer is pivotal. Another caveat is that we only consider a single-shot game. As argued by von der Fehr (2013), there is a risk that increased transparency in European electricity markets can facilitate tacit collusion in a repeated game.

Our study focuses on procurement auctions, but the results are analogous for multiunit sales auctions. Purchase constraints in sales auctions correspond to production capacities in our setting.[6] Analogous to the demand uncertainty in our model, the auctioneer's supply of securities is sometimes uncertain when bids are submitted in a multiunit sales auction.[7]

The remainder of the article is organized as follows. Section 2 compares details in our model with the previous literature. Section 3 formally introduces our model, which is analyzed for auctions with discriminatory and uniform pricing in Section 4. Section 5 summarizes our main results and their broader implications for auction market design. All proofs are in the Appendix.

2. Comparison with related studies

■ In this section, we discuss the major aspects of multiunit auction market design that our analysis builds on (i) bid constraints, (ii) asymmetric information, (iii) uniqueness of equilibria, (iv) comparisons of uniform-price and discriminatory auctions, and (v) pivotal producers in electricity markets.

□ **Related literature on bid constraints.** Electricity markets and other multiunit auctions often have restrictions on how many offer prices each producer can submit or, equivalently, how many steps a producer is allowed to have in its supply function. Similar to models of electricity markets by von der Fehr and Harbord (1993), Fabra, von der Fehr, and Harbord (2006), Fabra, von der Fehr, and de Frutos (2011) and Banal-Estanol and Micola (2009), we make the simplifying assumption that offers must be flat; a producer must offer its entire production capacity at the same price. In addition, we generalize the setting to cases where production costs are uncertain and asymmetric information among suppliers. Our model also generalizes Parisio and Bosco (2003), which is restricted to producers with flat offers and independent private costs in uniform-price auctions.

More generally, our analysis of symmetric BNE could also be applied to situations where each producer has multiple production plants and chooses a different offer price for each plant. This extension is straightforward if the market uncertainty is so small that each producer has only one plant (the same plant) that can be price-setting (marginal) with a positive probability in

[6] As an example, US treasury auctions have the 35% rule, which prevents a single bidder from buying more than 35% of the securities sold. Similar rules are used in spectrum auctions by the Federal Communications Commission (FCC) and in California's auctions of greenhouse gas emission allowances.

[7] In Mexico, Finland, and Italy, the treasury sometimes reduces the quantity of issued bonds after the bids have been received (McAdams, 2007). In treasury auctions in the United States, there is often an uncertain amount of noncompetitive bids from many small nonstrategic investors (Wang and Zender, 2002; Rostek, Weretka, and Pycia, 2010). Initial Public Offerings (IPO)s sometimes incorporate the so-called Greenshoe Option, which allow issuing firms to increase the amount of shares being offered by up to 15% after the bids have been submitted (McAdams, 2007).

equilibrium.[8] Hence, there is essentially only one offer per producer that matters.[9] A problem with such a multiplant extension is that nonuniqueness of equilibria becomes an issue when market uncertainties are small in a uniform-price auction. For example, in a related multiplant extension for a uniform-price auction where costs that are common knowledge, Fabra, von der Fehr, and Harbord (2006) derive another type of equilibrium, the asymmetric high-price NE. Uniqueness should be less of an issue in a discriminatory auction.[10]

Our bid constraint makes the discriminatory auction identical to a Bertrand game with both uncertain demand and costs. Thus, our framework encompasses the Bertrand models by Gal-Or (1986) and Spulber (1995), which consider producers with independent private costs.

In order to facilitate comparisons with previous studies, we are also interested in results for the limit where the cost uncertainty decreases until the costs are common knowledge. For producers that are both nonpivotal with certainty, we get the competitive outcome with zero markups, both for uniform and discriminatory pricing. This result agrees with the competitive outcomes for nonpivotal producers in von der Fehr and Harbord (1993) and in Fabra, von der Fehr, and Harbord (2006). If signals are independent and both producers are pivotal, it follows from Harsanyi's (1973) Purification Theorem that in the limit when costs are common knowledge, our BNE for uniform-price and discriminatory auctions correspond to the mixed-strategy NE analyzed by Anderson, Holmberg, and Philpott (2013), Anwar (2006), Fabra, von der Fehr, and Harbord (2006), Genc (2009), Son et al. (2004), and von der Fehr and Harbord (1993). Analogous mixed-strategy NE also occur in the Bertrand-Edgeworth game, when at least one producer is pivotal (Edgeworth, 1925; Allen and Hellwig, 1986; Beckmann, 1967; Levitan and Shubik, 1972; Maskin, 1986; Vives, 1986; Deneckere and Kovenock, 1996; Osborne and Pitchik, 1986).

Another consequence of the bid constraint is that uniform and discriminatory pricing are equivalent when both producers are nonpivotal with certainty, that is, when the capacity of each producer is always larger than realized demand. Independent of the auction format, the payoff is then zero for the producer with the highest offer price, and the other producer is paid its own offer price. This corresponds to the first-price single-object auction that is studied by Milgrom and Weber (1982). We generalize their model to the case where producers are pivotal with a positive probability, as can be the case in electricity markets.

□ **Asymmetric information in multiunit auctions.** Similar to us, Ausubel et al. (2014) and Vives (2011) consider multiunit auctions with asymmetric information, but they focus on producers that are nonpivotal with certainty. Moreover, we differ from them because we assume that offers must be flat.

If producers have asymmetric information about flat marginal costs, then it follows from Ausubel et al. (2014) that auctions can only be efficient if offers are also flat. The reason is that an efficient auction must accept the whole capacity of the low-cost bidder before any supply is accepted from the high-cost bidder, even if the difference in their realized flat marginal cost is arbitrarily small. Ausubel et al. (2014) identify special cases where unconstrained equilibrium offers are flat and allocations are efficient for the discriminatory auction. They also show that a uniform-price auction with unconstrained offers is generically inefficient. We require offers to be flat, which results in efficient auction outcomes (given aggregated market information) for symmetric equilibria in markets with inelastic demand and flat marginal costs, both for uniform

[8] Let n denote the production plant that is marginal/price-setting with a positive probability. As long as offers from units $u \geq n+1$ are not accepted in equilibrium, all of those units can be offered at the lowest marginal cost realization of the unit $n+1$, which would give a reservation price for unit n.

[9] Note that the discriminatory auction is different in that all accepted offers are price-setting (paid as bid). Still, it would be optimal for a producer to submit all offers that are certain to be accepted at the same price as the potentially marginal plant.

[10] A discriminatory multiplant auction should have a unique BNE if the highest marginal cost realization of the potentially marginal plant is equal to the lowest marginal cost realization for the next plant in the merit order, which corresponds to a reservation price.

and discriminatory pricing. Typically, unconstrained equilibrium offers in discriminatory auctions would be flatter (more elastic with respect to the price) than in uniform-price auctions (Genc, 2009; Anderson, Holmberg, and Philpott, 2013; Ausubel et al., 2014). Therefore, we conjecture that our bid constraint will have a greater positive influence on market performance in uniform-price auctions.

Related to the above, the results in Vives (2011) illustrate that the lack of bid constraints can have anticompetitive consequences in uniform-price auctions. In an auction where the costs are affiliated, a high clearing price is bad news for a firm's costs, because this increases the probability that the competitor has received a high-cost signal. Ausubel (2004) refers to this as the *Generalized winner's curse* or *Champion's plague*. As illustrated by Vives (2011), a producer therefore has an incentive to reduce its output when the price is unexpectedly high and increase its output when the price is unexpectedly low. This will make supply functions steeper or even downward sloping in auctions with nonrestrictive bidding formats, and this will significantly harm competition. If costs have a large common uncertainty, then markups in a uniform-price auction can be as high as for the monopoly case (Vives, 2011). Our restrictive bidding format avoids this problem. The bid constraint gives a producer less flexibility to condition its output on the competitor's information, and less possibilities to hedge the offer with respect to the Generalized winner's curse. It does not matter how sensitive a producer's cost is to the competitor's signal, our results are essentially the same irrespective of whether the costs are private, common, or anything in between those two extremes. Related results have been found in the literature on single-object auctions (Milgrom and Weber, 1982).

□ **Uniqueness of equilibria.** As, for example, illustrated by Wilson (1979), Klemperer and Meyer (1989), Green and Newbery (1992), and Ausubel et al. (2014), there are normally multiple NE in divisible-good auctions when some offers are never price-setting. The bid constraint mitigates this problem. In our setting, there is a unique equilibrium in the discriminatory auction also for a given demand level and given production capacities. In the uniform-price auction we get uniqueness, unless both producers are pivotal with certainty. Our uniqueness results have some parallels with Lizzeri and Perisco (2000). They find that uniqueness can normally only be ensured in single-object auctions where the payoff of the winning supplier is strictly increasing in its offer.[11] In our setting, this condition is always satisfied for the discriminatory auction. It is also satisfied for the uniform-price auction, unless both producers are pivotal with certainty.

Uniqueness of equilibria is another reason why highly anticompetitive equilibria in uniform-price auctions can be avoided. In the special case where both producers are pivotal with certainty, there is, in addition to the symmetric BNE that we calculate, also an asymmetric high-price equilibrium (von der Fehr and Harbord, 1993) in the uniform-price auction. This equilibrium is very unattractive for consumers of electricity, because the highest offer, which sets the clearing price, is always at the reservation price.[12]

□ **Comparisons of uniform-price and discriminatory auctions.** The bid constraint and the assumption that the pivotal status of producers is uncertain give a unique equilibrium. As far as we know, we are the first to compare designs of multiunit auctions with asymmetric information for settings with unique equilibria. In our setting, we find that an auctioneer would prefer uniform to discriminatory pricing if signals are affiliated, which is related to Milgrom and Weber's (1982) results for first- and second-price single-object auctions. We believe that our ranking of multiunit auctions is partly driven by our bid constraint, which seems to be particularly beneficial for

[11] Lizzeri and Perisco (2000) consider a sales auction, so actually their uniqueness condition is that the payoff of the winning bidder should be strictly decreasing in its bid. Lizzeri and Perisco (2000) consider a general single-object auction, where the loser could also get a payoff, but the payoff of the loser is restricted to be nonpositive. In our setting, the loser would also get a positive payoff in equilibrium.

[12] The equilibrium offer from the low-price bidder must be sufficiently low to ensure that the high-price bidder would not find it profitable to deviate and undercut the low-price bidder.

uniform-price auctions. Previous studies suggest that rankings of multiunit auctions become more ambiguous if one does not require offers to be flat. Holmberg (2009) and Hästö and Holmberg (2006) identify circumstances where discriminatory pricing is preferable to uniform pricing from the auctioneer's perspective. Pycia and Woodward (2015) identify circumstances when the auctions are equivalent. Ausubel et al. (2014) show that uniform pricing can be better or worse for the auctioneer. Our assumption that the pivotal status of producers is uncertain means that the high-price equilibrium can be avoided in the uniform-price auction. Otherwise, if this equilibrium exists and is selected by producers, then the auctioneer would prefer discriminatory pricing (Fabra, von der Fehr, and Harbord, 2006). Empirical studies by Armantier and Sbaï (2006,2009) and Hortaçsu and McAdams (2010) find that the treasury would prefer uniform pricing in France and Turkey, respectively, whereas Kang and Puller (2008) find that discriminatory pricing would be best for the treasury in South Korea. Given that our results suggest that details in the bidding format are important for the ranking of auctions, we think that future empirical rankings of multiunit auction designs could potentially benefit from considering even more details in the bidding format, as in the structural models of bidding behavior by Wolak (2007) and Kastl (2012).

□ **Pivotal producers in electricity markets.** In practice, the number of pivotal producers in wholesale electricity markets depends on the season and the time-of-day (Genc and Reynolds, 2011), but also on market shocks. Pivotal status indicators as measures of the ability to exercise unilateral market power have been evaluated by Bushnell, Knittel, and Wolak (1999) and Twomey et al. (2005) and have been applied by the Federal Energy Regulator Commission (FERC) in its surveillance of electricity markets in the United States. Such binary indicators are supported by von der Fehr and Harbord's (1993) high-price equilibrium in uniform-price auctions, where the market price is either at the marginal cost of the most expensive supplier or the reservation price, depending on whether producers are nonpivotal or pivotal with certainty. Our equilibrium is more subtle, the pivotal status can be uncertain before offers are submitted, and the expected market price increases continuously when producers are expected to be pivotal with a larger margin. Thus, in our setting, a high pivotal bidder frequency (PBF) is not a problem by itself, unless there are outcomes where a producer is pivotal by a large margin.

3. Model

■ Our model has two risk-neutral producers. They are symmetric *ex ante*, before each producer $i \in \{1, 2\}$ receives a private signal $s_i \in [\underline{s}, \overline{s}]$ with imperfect cost information. The joint probability density of signals $\chi(s_i, s_j)$ is continuously differentiable and symmetric, so that $\chi(s_i, s_j) \equiv \chi(s_j, s_i)$. Moreover, $\chi(s_i, s_j) > 0$ for $(s_i, s_j) \in (\underline{s}, \overline{s}) \times (\underline{s}, \overline{s})$.[13]

As in von der Fehr and Harbord (1993), we consider the case when each firm's marginal cost is flat up to its production capacity constraint $\overline{q}_i$.[14] However, in our setting, marginal costs are uncertain when offers are submitted. This is a realistic assumption for electricity markets, where real-time weather conditions can impact the thermal efficiency of fossil-fuel generation units or the opportunity cost of water for a hydroelectric, facility. We refer to $c_i(s_i, s_j)$ as the marginal cost of producer i, but costs are not necessarily deterministic, given s_i and s_j. More generally, $c_i(s_i, s_j)$ is the expected marginal cost conditional on all information available among producers in the market, so that

$$c_i\left(s_i, s_j\right) = \mathbb{E}\left[\tilde{c}_i | s_i, s_j\right],$$

where $\tilde{c}_i$ is the realized marginal cost of producer i. We assume that

$$\frac{\partial c_i\left(s_i, s_j\right)}{\partial s_i} > 0, \tag{1}$$

[13] We do not require $\chi(s_i, s_j) > 0$ at the boundary, but $\frac{\chi_1(u,\overline{s})}{\chi(u,\overline{s})} = \frac{\chi_2(\overline{s},u)}{\chi(\overline{s},u)}$ is assumed to be bounded for $u \in [\underline{s}, \overline{s}]$.

[14] This corresponds to flat demand in the sales auction of Ausubel et al. (2014).

so that the marginal cost of firm i increases with respect to its own signal s_i. We also require that the firm's cost is weakly increasing with respect to the competitor's signal s_j:

$$\frac{\partial c_i\left(s_i, s_j\right)}{\partial s_j} \geq 0. \tag{2}$$

A firm's private signal has more influence on its own cost than on the competitor's cost[15]:

$$\frac{\partial c_i\left(s_i, s_j\right)}{\partial s_i} > \frac{\partial c_j\left(s_j, s_i\right)}{\partial s_i}. \tag{3}$$

Taken together, (1) and (2) imply that:

$$\frac{dc_i\left(s, s\right)}{ds} > 0. \tag{4}$$

The special case with independent signals and $\frac{\partial c_i(s_i,s_j)}{\partial s_j} = 0$ corresponds to the private independent cost assumption, which is used in the analysis by Parisio and Bosco (2003) and Spulber (1995). Wilson (1979) uses a common cost/value assumption. Our model approaches this case in the limit where $\frac{\partial c_i(s_i,s_j)}{\partial s_i} - \frac{\partial c_j(s_j,s_i)}{\partial s_i} \searrow 0$.

Costs are insensitive to both signals in the limit when costs are common knowledge. For our BNE, it turns out that bidding behavior is determined by properties of the cost function along its diagonal, where producers receive identical private information. Thus, for us, it is sufficient to define a weaker form of common knowledge about costs. Let $\underline{c} = c_i(\underline{s}, \underline{s})$.

Definition 1. Production costs are insensitive to common variations in signals in the limit where $c_i(s, s) \searrow \underline{c}$ for $s \in [\underline{s}, \overline{s})$.

The production capacity $\overline{q}_i$ of a firm could be certain, as in von der Fehr and Harbord (1993), but we also allow $\overline{q}_i$ to be uncertain when offers are submitted. Uncertain production capacities would be consistent with the fact that generation units can have partial or complete outages after offers are first submitted and before units are called to operate. The production capacities of the two producers could be correlated, but they are symmetric information and we assume that they are independent of production costs and signals.[16] Capacities are symmetric *ex ante*, so that $\mathbb{E}[\overline{q}_i] = \mathbb{E}[\overline{q}_j]$. Realized production capacities are assumed to be observed by the auctioneer when the market is cleared.[17] This assumption is consistent with a must-offer requirement (which exists in most US wholesale markets) that requires a supplier to offer all available capacity in the wholesale market at or below the reservation price.

As in von der Fehr and Harbord (1993), demand is inelastic up to a reservation price $\overline{p}$. Moreover, demand is uncertain with compact support, $D \in [\underline{D}, \overline{D}]$. It could be correlated with the production capacities, but demand is assumed to be independent of the production costs and signals. In addition, it is assumed that all outcomes are such that $0 \leq D \leq \overline{q}_i + \overline{q}_j$, so that there is always enough production capacity in the market to meet the realized demand. Many wholesale electricity markets have a long-term resource adequacy process that makes sure that this condition holds. We say that producer $i \neq j$ is pivotal for outcomes where $D > \overline{q}_j$. Thus, a pivotal producer will have a strictly positive output irrespective of whether its offer is highest or lowest. Otherwise, the producer is nonpivotal. Due to uncertainties in demand and/or production capacities, the pivotal status of a producer is generally uncertain when it submits its offer.

[15] Note that we use the convention that a firm's own signal is placed first in its list of signals.

[16] In Europe, this assumption could be justified by the fact that any insider information on production capacities must be disclosed to the market according to EU No. 1227/2011 (REMIT).

[17] Alternatively, similar to the market design of the Australian wholesale market, producers could first choose bid prices and later adjust the quantity increments associated with each bid price just before the market is cleared. We assume that the reported production capacities are publicly verifiable, so that bidders cannot choose them strategically.

Analogous to Milgrom and Weber (1982), we assume that the reservation price is set at the smallest price, which ensures that there is production, that is, $\overline{p} = c_i(\overline{s}, \overline{s})$ for $i \in \{1, 2\}$. Note that we stick to this assumption even in the limit where production costs are insensitive to common variations in signals. Thus in this limit, $c_i(s, s)$ would jump from $\underline{c}$ to $\overline{p}$ as s approaches $\overline{s}$.

A firm $i \in \{1, 2\}$ submits its offer after it has received its private signal s_i. We assume that the bidding format constrains offers such that each firm must offer its entire production capacity at one unit price $p_i(s_i)$. The auctioneer accepts offers in order to minimize its procurement cost. Thus, output from the losing producer, which has the highest offer price, is only accepted when that firm is pivotal, and in that case, the auctioneer first accepts the entire production capacity from the winning producer, which has the lowest offer price.

Ex post, we denote the winning producer, which gets a high expected output, by subscript H. The losing producer, which gets a low expected output, is denoted by the subscript L. Winning and losing producers have the following expected outputs[18]:

$$q_H = \mathbb{E}\left[\min\left(\overline{q}_H, D\right)\right] \tag{5}$$

and

$$q_L = \mathbb{E}\left[\max\left(0, D - \overline{q}_H\right)\right]. \tag{6}$$

Given signals, the expected payoff of each producer is given by its expected revenue minus its expected production cost.

$$\pi_L = \left(p_L - c_L\left(s_L, s_H\right)\right) q_L$$
$$\pi_H = \left(p_H - c_H\left(s_H, s_L\right)\right) q_H.$$

Accepted offers in a discriminatory auction are paid as bid, that is, p_H is equal to the offer of the winning producer and p_L is equal to the offer of the losing producer. In the uniform-price auction, we have that $p_H = p_L$, and this price is set by the losing producer if it is pivotal. Otherwise, p_H is set by the offer of the winning producer, and no supply is accepted from the losing firm.

We assume that producers are risk-neutral and that each producer chooses its offer in order to maximize its expected profit, given its signal. We solve for the BNE in a one-shot game, where $p_i(s_i)$ is weakly monotonic and piece-wise differentiable.

In our analysis, we make use of the concept affiliated signals. Signals are affiliated when

$$\frac{\chi\left(u, v'\right)}{\chi\left(u, v\right)} \leq \frac{\chi\left(u', v'\right)}{\chi\left(u', v\right)}, \tag{7}$$

where $v' \geq v$ and $u' \geq u$. Thus, if the signal of one player increases, then it (weakly) increases the probability that its competitor has a high signal relative to the probability that its competitor has a low signal. We say that signals are negatively affiliated when the opposite is true, that is,

$$\frac{\chi\left(u, v'\right)}{\chi\left(u, v\right)} \geq \frac{\chi\left(u', v'\right)}{\chi\left(u', v\right)}, \tag{8}$$

where $v' \geq v$ and $u' \geq u$. Note that independent signals are both affiliated and negatively affiliated. We let

$$F\left(s_i\right) = \int_{-\infty}^{s_i} \int_{-\infty}^{\infty} \chi\left(u, v\right) dv du$$

[18] A rationing rule is used when producers submit offers at the same price and realized demand is strictly less than the realized market capacity. The details of the rule does not influence our results, as long as the rationing rule is such that, whenever rationing is needed, any producer would get a significantly larger output, an increment bounded away from zero, if it reduced its offer price by any positive amount. As an example, the pro-rata-on-the-margin rule, which is a standard rationing rule in multiunit auctions (Kremer and Nyborg, 2004), would satisfy these properties, and so would the large class of disproportionate rationing rules that is considered by Holmberg (2017).

denote the marginal distribution, that is, the unconditional probability that supplier i receives a signal below s_i. Moreover, we define the marginal density of firm i's signal as:

$$f(s_i) = F'(s_i).$$

4. Analysis

□ **Discriminatory pricing.** We start the section on discriminatory pricing by deriving the best response of firm $i \in \{1, 2\}$. We denote the competitor by $j \neq i$. Recall that each firm is paid as bid under discriminatory pricing and demand uncertainty, and the production capacity uncertainties are independent of the cost uncertainties. Thus, the expected profit of firm i when receiving signal s_i is:

$$\pi_i(s_i) = \left(p_i(s_i) - \mathbb{E}\left[\tilde{c}_i \,|\, s_i, p_j \geq p_i\right]\right) \Pr\left(p_j \geq p_i \middle| s_i\right) q_H + \left(p_i(s_i) - \mathbb{E}\left[\tilde{c}_i \,|\, s_i, p_j \leq p_i\right]\right)\left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L. \tag{9}$$

In the Appendix, we show that:

Lemma 1. In markets with discriminatory pricing:

$$\frac{\partial \pi_i(s_i)}{\partial p_i} = \underbrace{\Pr\left(p_j \geq p_i \middle| s_i\right) q_H + \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L}_{\text{price effect}} + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right) \underbrace{\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}(q_H - q_L)}_{\text{quantity effect}}, \tag{10}$$

whenever $p_i(s_i)$ and $p_j(s_j)$ are locally differentiable and locally invertible for signals that have offer prices near $p_i(s_i)$.

In the Appendix and in the proof of Proposition 1, we show that equilibrium offers need to be locally differentiable and invertible, so that (10) must necessarily hold in equilibrium. The first two terms on the right-hand side of (10) correspond to the price effect. This is what the producer would gain in expectation from increasing its offer price by one unit if the acceptance probabilities were to remain unchanged. However, on the margin, a higher offer price lowers the probability of being the winning producer by $\frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i}$. Switching from being the winning to the losing bidder reduces the accepted quantity by $q_H - q_L$. We refer to $\frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i}(q_H - q_L)$ as the quantity effect, that is, the quantity that is lost on the margin from a marginal price increase. The markup for lost sales, $p_i - c_i(s_i, p_j^{-1}(p_i))$, times the quantity effect gives the lost value of the quantity effect. This is the last term on the right-hand side of (10). Note that marginal changes in $p_i(s_i)$ only result in changes in output for cases where the competitor, producer j, is bidding very close to p_i, which corresponds to the competitor receiving the signal $p_j^{-1}(p_i)$. This explains why $c_i(s_i, p_j^{-1}(p_i))$ is the relevant marginal cost in the markup for lost sales in the quantity effect.

We find it useful to introduce the function $H^*(s)$, which is proportional to the quantity effect and inversely proportional to the price effect for a given signal s.

Definition 2.

$$H^*(s) := \frac{\chi(s, s)(q_H - q_L)}{\int_s^{\bar{s}} \chi\left(s, s_j\right) ds_j q_H + \int_{\underline{s}}^{s} \chi\left(s, s_j\right) ds_j q_L}. \tag{11}$$

$H^*(s)$ depends on exogenous variables/parameters and captures the essential aspects of the information structure, the auction format, and the essential properties of demand and the production capacities. The unique equilibrium is symmetric, so in the following, we sometimes find it convenient to drop subscripts.

Proposition 1. If

$$\frac{d}{ds}\left(\frac{\int_x^{\bar{s}} \chi\left(s, s_j\right) ds_j q_H + q_L \int_{\underline{s}}^x \chi\left(s, s_j\right) ds_j}{\chi\left(s, x\right)}\right) \geq 0, \tag{12}$$

for all $s, x \in (\underline{s}, \bar{s})$, then there is a unique BNE in the discriminatory auction. The unique equilibrium is symmetric, efficient (given aggregated market information), and has the property that $p'(s) > 0$, where

$$p(s) = c\left(s, s\right) + \int_s^{\bar{s}} \frac{dc\left(v, v\right)}{dv} e^{-\int_s^v H^*(u)du} dv \tag{13}$$

for $s \in [\underline{s}, \bar{s})$. The above expression can be simplified for the following circumstances:

(1) In the limit when production costs are insensitive to common variations in signals, (13) can be simplified to:

$$p(s) = \underline{c} + e^{-\int_s^{\bar{s}} H^*(u)du}\left(\bar{p} - \underline{c}\right). \tag{14}$$

(2) The condition in (12) is satisfied if signals are independent, and (13) can then be simplified to:

$$p(s) = c\left(s, s\right) + \int_s^{\bar{s}} \frac{dc\left(v, v\right)}{dv}\left(\frac{(1 - F\left(v\right)) q_H + F\left(v\right) q_L}{(1 - F(s)) q_H + F(s) q_L}\right) dv. \tag{15}$$

In the limit when production costs are insensitive to common variations in signals, then (15) can be further simplified to:

$$p(s) = \underline{c} + \left(\frac{q_L}{((1 - F\left(s\right)) q_H + F(s) q_L)}\right)\left(\bar{p} - \underline{c}\right). \tag{16}$$

(3) The condition in (12) is satisfied if signals are affiliated and both producers are nonpivotal with certainty, which implies $q_L = 0$, in which case (11) can be simplified to:

$$H^*(s) := \frac{\chi\left(s, s\right)}{\int_s^{\bar{s}} \chi\left(s, s_j\right) ds_j}. \tag{17}$$

If, in addition, production costs are insensitive to common variations in signals, the equilibrium offer is perfectly competitive, that is, $p(s) = \underline{c}$ for $s \in [\underline{s}, \bar{s})$.

Demand is inelastic, so the total output is efficient. The marginal costs are flat and in our unique equilibrium, the firm with the highest cost and signal makes the highest offer. Thus, the bid constraint ensures that the allocation is efficient (given aggregated market information).

Another conclusion that we can draw from Proposition 1 is that bidding behavior is only influenced by properties of $c_i(s_i, s_j)$ at points where $s_i = s_j$. Thus, for given properties along the diagonal of the cost function, where signals are identical, it does not matter for our analysis whether the costs are private, so that $\frac{\partial c_i(s_i, s_j)}{\partial s_j} = 0$, or whether the costs have a common uncertainty component, such that $\frac{\partial c_i(s_i, s_j)}{\partial s_j} > 0$. As noted above, the reason is that when solving for the locally optimal offer price, a producer is only interested in cases where the competitor is bidding close to p_i. In a symmetric equilibrium, this occurs when the competitor receives a similar signal. The properties of the function $c(\cdot, \cdot)$ for signals where $s_i \neq s_j$ could influence the expected production cost of a firm, but not its bidding behavior. There are related results for single-object auctions

(Milgrom and Weber, 1982). However, the outcome would be different if each producer submitted an offer with multiple offer prices or even a continuous supply function as in Vives (2011), so that a producer could indirectly condition its output on the competitor's information.

The second-order condition in (12) makes sure that firm i has an incentive to increase its offer price, the ratio of the quantity and price effects becomes small, if producer i has chosen a low offer (below the symmetric equilibrium) such that $p_i(s) = p_j(x)$ for $s > x$. Similarly, the condition gives firm i an incentive to decrease its offer if it has chosen offers above the symmetric equilibrium such that $p_i(s) = p_j(x)$ for $s < x$.

It follows from Definition 2 and Proposition 1 that $H^*(s)$ and $p(s)$ are determined by the expected sales of the high-price supplier, q_H, and the expected sales of the low-price supplier, q_L, but $H^*(s)$ and $p(s)$ are independent of the variances of those sales. Moreover,

Proposition 2. In a discriminatory auction, *offers become more competitive, $p(s)$ decreases for every $s \in [\underline{s}, \overline{s})$, if $H^*(s)$ increases for every $s \in (\underline{s}, \overline{s})$, which is the case if $q_H \geq q_L > 0$* and q_H *increases and/or q_L decreases.*

As an example, q_H would increase and q_L decrease, if the size of producers' capacities were increased, unless producers are nonpivotal with certainty.[19] Recall that $H^*(s)$ is proportional to the quantity effect and inversely proportional to the price effect, so it makes sense that a high $H^*(s)$ results in more competitive offers with lower markups.

Costs that are common knowledge constitute a special case of the limit where firms' marginal costs are insensitive to common variations in signals, as in (14). If costs are common knowledge, the signals only serve the purpose of coordinating producers' actions, as in a correlated equilibrium (Osborne and Rubinstein, 1994). If, in addition, signals are independent as in (16), signals effectively become randomization devices of a mixed-strategy NE. To illustrate this, signals could be transformed from s to $P = p(s)$, that is, a signal that directly gives the offer price that a firm should choose. The price signal has the probability distribution $G(P) = F(p^{-1}(P))$. If we rewrite (16), we get that

$$G(P) = \frac{q_H}{q_H - q_L} - \frac{q_L(\overline{p} - \underline{c})}{(q_H - q_L)(P - \underline{c})}. \tag{18}$$

This probability distribution of offer prices corresponds to the mixed-strategy NE that is calculated for discriminatory auctions by Fabra, von der Fehr, and Harbord (2006). This confirms Harsanyi's (1973) Purification Theorem, that a mixed-strategy NE is equivalent to a pure-strategy BNE, where costs are common knowledge and signals are independent.

If producers have sufficiently large capacity realizations, so that $\overline{q}_i > D$ for both producers and all outcomes, then both producers are nonpivotal with certainty as in Case 3 of Proposition 1, and only the lowest (winning) offer is accepted for every outcome. In this special case, there is no difference between the discriminatory and uniform-price auction in our setting, because the winning offer sets its own price also in the uniform-price auction. This case also corresponds to the first-price single-object auction, which is analyzed by Milgrom and Weber (1982). As in Milgrom and Weber (1982), private information normally gives an informational rent, so if costs are asymmetric information, then suppliers have a positive markup, even if they are both nonpivotal with certainty. However, the markup $p(s) - c(s, s)$ is zero in the limit when production costs are insensitive to common variations in signals. For the special case where costs are common knowledge, this result concurs with the results for uniform-price and discriminatory auctions by von der Fehr and Harbord (1993) and Fabra, von der Fehr, and Harbord (2006). Finally, recall that our discriminatory auction is identical to the Bertrand model, so our results for the discriminatory auction also apply to the Bertrand game. In particular, Case 3 with nonpivotal

[19] Analogously, less restrictive purchase constraints in a sales auction would make bidding more competitive.

producers corresponds to the classical Bertrand game, whereas producers are typically pivotal in the Bertrand-Edgeworth game.

□ **Uniform pricing.** As mentioned earlier, Case 3 in Proposition 1 also applies when both producers are nonpivotal with certainty in a uniform-price auction. Now, we consider the other extreme, where both producers are pivotal with certainty in a uniform-price auction. As in the discriminatory auction, we solve for a symmetric equilibrium. Later, we will consider the general case where the pivotal status of producers is uncertain in the uniform-price auction, in which case the unique equilibrium is symmetric.

The highest offer sets the market price in a uniform-price auction when both producers are pivotal with certainty. The demand and production capacity uncertainties are independent of the signals and cost uncertainties. Thus, when producers are both pivotal with certainty, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned}\pi_i(s_i) = \mathbb{E}\left[p_j - \tilde{c}_i \middle| p_j \geq p_i; s_i\right] \Pr\left(p_j \geq p_i \middle| s_i\right) q_H \\ + \left(p_i(s_i) - \mathbb{E}\left[\tilde{c}_i | p_j \leq p_i; s_i\right]\right)\left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L.\end{aligned} \tag{19}$$

Lemma 2. In a uniform-price auction with producers that are both pivotal with certainty, we have:

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} = \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L \\ + \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\left(q_H - q_L\right),\end{aligned} \tag{20}$$

whenever $p_i(s_i)$ and $p_j(s_j)$ are locally differentiable and locally invertible for signals that have offer prices near $p_i(s_i)$.

The first-order condition for the uniform-price auction is similar to the first-order condition for the discriminatory auction in Lemma 1, but there is one difference. In contrast to the discriminatory auction, the lowest bidder does not gain anything from increasing its offer price in a uniform-price auction when both producers are pivotal with certainty, assuming it is strictly below the highest bidder. Thus, the price effect has one term less in the uniform-price auction, which reduces the price effect. There is a corresponding change in the H function, which is proportional to the quantity effect and inversely proportional to the price effect.

$$\hat{H}(s) = \frac{(q_H - q_L)\,\chi\,(s, s)}{q_L \int_{\underline{s}}^{s} \chi\left(s, s_j\right) ds_j}. \tag{21}$$

Proposition 3. The symmetric BNE offer in a uniform-price auction where both producers are pivotal with certainty is given by

$$p(s) = c\,(s, s) + \int_s^{\overline{s}} \frac{dc\,(v, v)}{dv} e^{-\int_s^v \hat{H}(u)du} dv\ , \tag{22}$$

for $s \in [\underline{s}, \overline{s})$ if signals are negatively affiliated. The equilibrium is efficient (given aggregated market information) and has the property that $p'(s) > 0$. The expression can be simplified for the following circumstances:

(1) In the limit when production costs are insensitive to common variations in signals, (22) can be simplified to:

$$p(s) = \underline{c} + e^{-\int_s^{\overline{s}} \hat{H}(u)du}\left(\overline{p} - \underline{c}\right). \tag{23}$$

(2) Independent signals are negatively affiliated. In this case, (22) simplifies to:

$$p(s) = c(s, s) + \int_{s}^{\overline{s}} \frac{dc(v, v)}{dv} \left(\frac{F(s)}{F(v)} \right)^{\frac{(q_H - q_L)}{q_L}} dv. \tag{24}$$

In the limit when production costs are insensitive to common variations in signals, then (24) can be further simplified to

$$p(s) = \underline{c} + (F(s))^{\frac{(q_H - q_L)}{q_L}} (\overline{p} - \underline{c}). \tag{25}$$

We can use an argument similar to the one we used for the discriminatory auction to show that the limit result in (25) corresponds to the mixed-strategy NE that is derived for uniform-price auctions by von der Fehr and Harbord (1993). (25) can also be used to calculate the expected clearing price.

Proposition 4. If the signals are independent, the production costs are insensitive to common variations in signals, and both producers are pivotal with certainty, then the expected market price for the symmetric equilibrium in the uniform-price auction is given by:

$$\overline{p} - \frac{(\overline{p} - \underline{c})(q_H - q_L)}{q_H + q_L}.$$

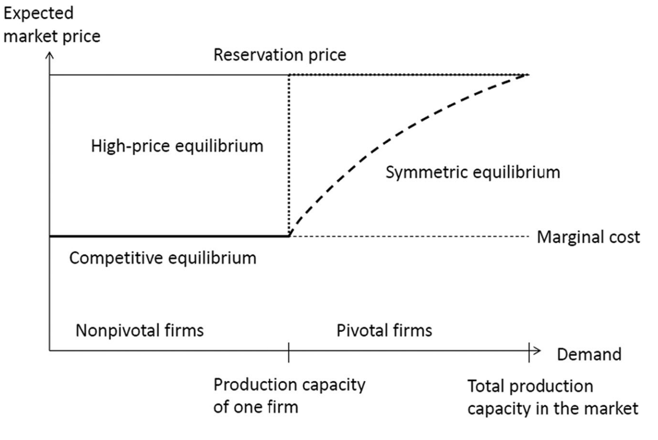
In the special case with certain demand and certain production capacities, such that both producers are pivotal, we have $q_H = \overline{q}$ and $q_L = D - \overline{q} > 0$, so that the expected market price is given by

$$\overline{p} - \frac{(\overline{p} - \underline{c})(2\overline{q} - D)}{D}. \tag{26}$$

Figure 1 plots this relationship, which gives a comparative statics analysis of the expected transaction price with respect to a certain demand level. The dashed line shows that the expected market price increases continuously as demand increases beyond the production capacity of one firm and it does not reach the reservation price until demand equals the total production capacity in the market. For an extension of our model with more than two firms in the market,

FIGURE 1

COMPARATIVE STATICS ANALYSIS FOR OUR SYMMETRIC EQUILIBRIUM AND VON DER FEHR AND HARBORD'S (1993) ASYMMETRIC HIGH-PRICE EQUILIBRIUM IN A UNIFORM-PRICE AUCTION WHERE PRODUCERS HAVE A CERTAIN PIVOTAL STATUS, COSTS ARE COMMON KNOWLEDGE, AND SIGNALS ARE INDEPENDENT



the expected price in our model would stay near the marginal cost until demand is near the total production capacity in the market, where the expected price will take off toward the reservation price. This would be reminiscent of what is often called "hockey-stick pricing" that is typical for wholesale electricity markets (Hurlbut, Rogas, and Oren, 2004; Holmberg and Newbery, 2010). In Figure 1, we also plot the high-price equilibrium in von der Fehr and Harbord (1993). In this equilibrium, the market price jumps directly from the competitive price with zero markups up to the reservation price when demand increases at the critical point where both producers switch from being nonpivotal to being pivotal with certainty in a uniform-price auction.

In the comparative statics analysis in Figure 1, where costs are common knowledge, the expected transaction price and payoffs are continuous at the point where both producers switch from being nonpivotal to pivotal. The same is true for the equilibrium offers at this point. When costs are common knowledge, both nonpivotal producers and just pivotal producers will make perfectly competitive offers. However, this will change for uncertain costs. It follows from Case 3 in Proposition 1 that $p(s) > c(s, s)$ for a uniform-price auction where both producers are nonpivotal with certainty, which corresponds to a first-price auction. If both producers are just pivotal with certainty, so that $q_L \searrow 0$, then it follows from Proposition 3 that $p(s) = c(s, s)$. This corresponds to Milgrom and Weber's (1982) results for second-price sales auctions, because the lowest bidder gets to produce (almost) the whole demand, whereas the highest bidder sets the uniform market price. Thus, the comparative statics analysis of offer prices has a discontinuity if demand increases at the critical point where both producers' capacities switch from being nonpivotal with certainty to being pivotal with certainty. Somewhat counterintuitively, offer prices decrease at this critical point, even if demand increases. On the other hand, the highest offer sets the price when producers are just pivotal. Thus, payoffs are typically positive when costs are uncertain, irrespective of the pivotal status of producers. It follows from the revenue-equivalence results for first- and second-price auctions that expected transaction prices and payoffs would be continuous at the critical point if signals are independent. However, it follows from Milgrom and Weber's (1982) ranking of first- and second-price auctions that the producers' revenues would shift downward at the critical point where both producers' capacities switch from being nonpivotal with certainty to being pivotal with certainty, if the signals of producers are affiliated.

□ **Uncertain pivotal status.** In the general case, the pivotal status of each producer is uncertain when its offer is submitted. In this case, both the lowest and highest offer are price-setting with some probability, and this will ensure a unique equilibrium. In particular, the asymmetric high-price equilibrium does not exist when the pivotal status of both producers is uncertain.[20]

In a uniform-price auction, the pivotal status of the highest bidder determines who sets the uniform price. If the highest bidder is nonpivotal, then the lowest offer sets the price, as in a discriminatory auction. We denote the probability for this event by Π^{NP}. If, on the other hand, the highest bidder is pivotal, then its offer sets the price. This means that the expected transaction price of the winning producer becomes a convex combination of the highest and lowest offer in the market. This has some similarities to the single-object sales auction studied by Plum (1992), where the pricing rule is such that the winner pays a convex combination of the highest and second-highest bid. One difference is that the losing producer would sometimes get a positive payoff (when it is pivotal) in our setting, whereas Plum (1992) assumes that the losing bidder always get a zero profit, which corresponds to the case that both producers are nonpivotal with certainty.

[20] Uncertain pivotal status implies that the lowest bidder will set its transaction price with a positive probability. As shown by von der Fehr and Harbord (1993), this implies that the lowest bidder would find it optimal to choose an offer just below the high-price offer at the reservation price. However, this means that the high-price bidder, would find it optimal to deviate and slightly undercut the low-price bidder.

Lemma 3. In a uniform-price auction, where $p_i(s_i)$ and $p_j(s_j)$ are locally differentiable and locally invertible, we have:

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} &= \Pr\left(p_j \geq p_i \middle| s_i\right) q_H^{NP} \Pi^{NP} + \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L \\ &+ \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)(q_H - q_L),\end{aligned} \tag{27}$$

where

$$q_H^{NP} = \mathbb{E}\left[D \middle| \overline{q}_H \geq D\right].$$

Thus, the quantity effect is similar to when producers are both pivotal with certainty. However, the price effect depends on the probability that the highest bidder is nonpivotal. Increasing an offer price only contributes to the price effect when a producer's offer is price-setting, that is, when the producer is pivotal and has the highest offer price or when the producer has the lowest offer price and the competitor is nonpivotal. The function $\hat{H}(s)$ generalizes as follows:

Definition 3.

$$\hat{H}(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi\left(s, s_j\right) ds_j q_H^{NP} \Pi^{NP} + \int_{\underline{s}}^{s} \chi\left(s, s_j\right) ds_j q_L}.$$

Proposition 5. If the pivotal status is uncertain and

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi\left(s, s_j\right) ds_j q_H^{NP} \Pi^{NP} + q_L \int_{\underline{s}}^{x} \chi\left(s, s_j\right) ds_j}{\chi(s, x)}\right) \geq 0 \tag{28}$$

for all $s, x \in (\underline{s}, \overline{s})$, then there is a unique BNE in the uniform-price auction. The unique equilibrium is symmetric, efficient, and has the property that $p'(s) > 0$, where:

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \hat{H}(u)du} dv, \tag{29}$$

for $s \in [\underline{s}, \overline{s})$. The expression can be simplified for the following circumstances:

(1) In the limit when production costs are insensitive to common variations in signals, (29) can be simplified to:

$$p(s) = \underline{c} + e^{-\int_s^{\overline{s}} \hat{H}(u)du} \left(\overline{p} - \underline{c}\right). \tag{30}$$

(2) Independent signals satisfy the condition in (28), in which case, (29) simplifies to:

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} \left(\frac{(1 - F(v)) q_H^{NP} \Pi^{NP} + F(v) q_L}{(1 - F(s)) q_H^{NP} \Pi^{NP} + F(s) q_L}\right)^{\frac{(q_H - q_L)}{q_H^{NP} \Pi^{NP} - q_L}} dv. \tag{31}$$

If, in addition to independent signals, production costs are insensitive to common variations in signals, then (31) can be simplified to

$$p(s) = \underline{c} + \left(\frac{q_L}{\left((1 - F(s)) q_H^{NP} \Pi^{NP} + F(s) q_L\right)}\right)^{\frac{(q_H - q_L)}{q_H^{NP} \Pi^{NP} - q_L}} \left(\overline{p} - \underline{c}\right). \tag{32}$$

From Definition 3 and Proposition 5, it can be shown that:

Proposition 6. In a uniform-price auction, *offers become more competitive,* $p(s)$ *decreases for every* $s \in [\underline{s}, \overline{s})$, *if* $\widehat{H}(s)$ *increases for every* $s \in (\underline{s}, \overline{s})$, *which is the case if* $q_H \geq q_L > 0$ and q_H *increases and/or* q_L *decreases.*

□ **Comparison of auction formats.** We know from Proposition 1 and Proposition 5 that production is efficient for both auction formats in our setting with inelastic demand, *ex ante* symmetry, monotonic equilibria, flat marginal cost, and flat offers. However, equilibrium offers will normally depend on the auction format. It follows from Definition 2 and Definition 3 that we have $\hat{H}(s) \geq H^*(s)$. Thus, the following conclusions can be drawn from Proposition 1 and Proposition 5.

Corollary 1. Assume that the pivotal status of both producers is uncertain, and that there exists a pure-strategy equilibrium in both auctions. For any signal $s_i \in [\underline{s}, \overline{s}]$, the equilibrium offer of producer $i \in \{1, 2\}$ is weakly higher in a discriminatory auction than in a uniform-price auction. Moreover, the highest accepted offer in the auction (the stop-out price) is always weakly higher in the discriminatory auction.

Even if offers are higher in a discriminatory auction, it does not necessarily mean that the expected payment to producers is higher, because the market price is set by the highest offer in a uniform-price auction. The following proposition identifies circumstances where the expected payment is lower in a uniform-price auction.

Proposition 7. Assume that signals of producers are affiliated, the pivotal status is uncertain for both producers, and that a symmetric pure-strategy BNE exists in both auctions, then the expected payment to producers is weakly higher in a discriminatory auction compared to a uniform-price auction. In the special case where producers observe independent signals, then the expected payment to producers is the same in both auctions (revenue equivalence).

The revenue-equivalence result for independent signals and uncertain pivotal status concurs with Fabra, von der Fehr, and Harbord (2006) who showed that expected payoffs are the same for uniform and discriminatory pricing when costs are common knowledge. Our ranking for affiliated signals is related to and generalizes some aspects of Milgrom and Weber's (1982) ranking of first- and second-price auctions in single-object auctions.

□ **Publicity effect.** In this subsection, we consider the case where the auctioneer has private information about production costs in the market, a signal y, and we analyze whether it would be beneficial for the auctioneer to disclose this signal to both producers. In an electricity market, this signal can, for example, be thought of as a measure of the opportunity cost of water in a hydro-dominated market or an estimate of the short-term price of natural gas in a gas-dominated market.

We assume that the auctioneer's signal is independent of demand and production capacities. We consider a symmetric equilibrium where a producer submits an offer $p^{\mathbb{C}}(s_i; y)$ after observing both its private signal s_i and the common signal y. A producer i has the expected marginal cost $c^{\mathbb{C}}(s_i, s_j; y)$ conditional on all information that is available in the market, that is,

$$c^{\mathbb{C}}\left(s_i, s_j; y\right) = \mathbb{E}\left[\tilde{c}_i \,|\, s_i, s_j, y\right].$$

We let $\chi^{\mathbb{C}}(s_i, s_j; y)$ be the joint probability density for signals of the producers conditional on the common signal y.

Assumption 1. A producer's marginal cost $c^{\mathbb{C}}(s_i, s_j; y)$ is nondecreasing with respect to all signals, strictly increasing with respect to its own signal, and such that $c^{\mathbb{C}}(\bar{s}, \bar{s}; y) = \bar{p}$.

The last part of the assumption implies that the upper bound on expected costs is large enough so that public information has no influence at that point.

Proposition 8. Consider a discriminatory auction where the auctioneer has disclosed a signal y to both producers and where Assumption 1 is satisfied. If

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi^{\mathbb{C}}\left(s, s_j; y\right) ds_j q_H + q_L \int_{\underline{s}}^{x} \chi^{\mathbb{C}}\left(s, s_j; y\right) ds_j}{\chi^{\mathbb{C}}\left(s, x; y\right)}\right) \geq 0, \tag{33}$$

for all $s, x \in (\underline{s}, \overline{s})$, then there is a unique BNE in the auction. The unique equilibrium is symmetric, efficient (given aggregated market information), and has the property that $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial s} > 0$, where

$$p^{\mathbb{C}}\left(s; y\right) = c^{\mathbb{C}}\left(s, s; y\right) + \int_s^{\overline{s}} \frac{dc^{\mathbb{C}}\left(v, v; y\right)}{dv} e^{-\int_s^v H^{\mathbb{C}}(u;y)du} dv \tag{34}$$

$$H^{\mathbb{C}}\left(s; y\right) = \frac{\chi^{\mathbb{C}}\left(s, s; y\right)\left(q_H - q_L\right)}{\int_s^{\overline{s}} \chi^{\mathbb{C}}\left(s, s_j; y\right) ds_j q_H + \int_{\underline{s}}^{s} \chi^{\mathbb{C}}\left(s, s_j; y\right) ds_j q_L}, \tag{35}$$

for $s \in [\underline{s}, \overline{s})$. Moreover, we have that $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$ if we assume that $\frac{\partial H^{\mathbb{C}}(u;y)}{\partial y} \leq 0$, or if $\frac{\partial c^{\mathbb{C}}(s,s;y)}{\partial y}$ is sufficiently large.

Disclosure of the common signal y will not change the allocation. In equilibrium, it is still the producer that receives the lowest private signal that will have the lowest offer. However, disclosing the signal y will influence the profits of producers.

Proposition 9. Consider a discriminatory auction which satisfies Assumption 1 and which has the following properties: i) all private and public signals are affiliated, ii) $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$, and iii) a pure-strategy equilibrium exists irrespective of whether the auctioneer's signal y is disclosed. In such an auction, disclosing the auctioneer's signal y to both producers will weakly reduce the procurement cost of the auctioneer.

Next, we will consider related results for the uniform-price auction.

Proposition 10. Consider a uniform-price auction with uncertain pivotal status, where the auctioneer has disclosed a signal y to both producers and where Assumption 1 is satisfied. If

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi\left(s, s_j; y\right) ds_j q_H^{NP} \Pi^{NP} + q_L \int_{\underline{s}}^{x} \chi\left(s, s_j; y\right) ds_j}{\chi\left(s, x; y\right)}\right) \geq 0 \tag{36}$$

for all $s, x \in (\underline{s}, \overline{s})$, then there is a unique BNE in the auction. The unique equilibrium is symmetric, efficient, and has the property that $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial s} > 0$, where

$$p^{\mathbb{C}}\left(s; y\right) = c^{\mathbb{C}}\left(s, s; y\right) + \int_s^{\overline{s}} \frac{dc^{\mathbb{C}}\left(v, v; y\right)}{dv} e^{-\int_s^v \widehat{H}^{\mathbb{C}}(u;y)du} dv \tag{37}$$

$$\widehat{H}^{\mathbb{C}}\left(s; y\right) = \frac{\chi^{\mathbb{C}}\left(s, s; y\right)\left(q_H - q_L\right)}{\int_s^{\overline{s}} \chi^{\mathbb{C}}\left(s, s_j; y\right) ds_j q_H^{NP} \Pi^{NP} + \int_{\underline{s}}^{s} \chi^{\mathbb{C}}\left(s, s_j; y\right) ds_j q_L}, \tag{38}$$

for $s \in [\underline{s}, \overline{s})$. Moreover, we have that $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$ if we assume that $\frac{\partial \widehat{H}^{\mathbb{C}}(u;y)}{\partial y} \leq 0$, or if $\frac{\partial c^{\mathbb{C}}(s,s;y)}{\partial y}$ is sufficiently large.

When proving the publicity effect for the discriminatory auction, we make use of a linkage-principle argument (Milgrom and Weber, 1982): the greater the linkage between a producer's private signal and his expected transaction price, the more competitive he will bid. We essentially show that if the auctioneer discloses y, then this strengthens the linkage. Intuitively, this should also be true for uniform-price auctions, but in this case, we can only show this when producers have independent signals.[21]

Proposition 11. Consider a uniform-price auction which satisfies Assumption 1 and has the following properties: i) pivotal status is uncertain, ii) producers have conditionally independent signals, possibly affiliated with the public signal, and iii) $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$. In such an auction, disclosing the auctioneer's signal y to both producers will weakly reduce the procurement cost of the auctioneer.

5. Concluding discussion

■ We have analyzed a stylized duopoly model of a divisible-good procurement auction with production uncertainty, which is of relevance for wholesale electricity markets. Each producer receives a private signal with imperfect cost information from a bivariate probability distribution (known to each producer) and then chooses one offer price for its entire production capacity. The demand and production capacities could also be uncertain. A producer is pivotal when the realized capacity of its competitor is smaller than realized demand. Marginal costs are flat (independent of output) up to the capacity constraint. We assume that the bidding format has the constraint that offers must also be flat.

Multiplicity of equilibria often occur in situations where some bids are never price-setting, which gives bidders freedom when choosing such bids. The bid constraint gives bidders less flexibility in such situations, and this facilitates uniqueness of equilibria under our bid restrictions. We find that there is a unique BNE, which is symmetric, in the discriminatory auction. The uniform-price auction has a unique equilibrium, which is symmetric, when the pivotal status of both producers is uncertain. The bid constraint also reduces production inefficiencies in our setting. The bid constraint mitigates Vives (2011) highly anticompetitive outcomes for uniform-price auctions where costs have large common uncertainties. This indicates that our bid constraint could potentially be beneficial for welfare and the auctioneer in uniform-price auctions of forward contracts and hydro-dominated electricity markets. This article does not explore this in detail, but it is our belief that the auctioneer would generally benefit from restricting offers to have a shape/slope that is similar to the shape/slope of the marginal cost. As an example, if each plant has a constant marginal cost independent of output, then we conjecture that it would be beneficial for an auctioneer to restrict the number of steps in the offer stack of a producer such that it is equal to the number of plants, as in the electricity market of Colombia. The parallel work by Anderson and Holmberg (2018) gives this conjecture some support. They consider a Colombian multiplant market where producers are symmetric *ex ante*, signals are independent, demand is inelastic and uncertain, and the cost uncertainty is small so that the merit order of plants stays the same. They find that there are no welfare losses in a multiunit auction under those circumstances.

When offers are constrained to be flat, we find that an auctioneer would prefer uniform pricing when the signals are affiliated. We show that equilibrium offers in a discriminatory auction are determined by the expected sales of the producer with the highest and lowest offer price, respectively. A smaller difference between these sales means that both producers are pivotal by a larger margin and equilibrium offers increase. The variance of these sales—due to demand shocks, production outages, and volatile renewable production—will not influence the bidding behavior of producers in a discriminatory auction. Bidding in the uniform-price auction is more sensitive to this variance. Still, expected payoffs are not that sensitive. For given expected sales of the highest and lowest bidder, the probability that a producer is pivotal in a uniform-price auction does not influence the expected payoffs if producers have independent signals. For affiliated

[21] One problem with the uniform-price auction, compared to discriminatory pricing, is that we need to assume that producers have independent signals to make sure that the expected transaction price of a producer does not increase with respect to its private signal when its offer is kept fixed, unless the auctioneer discloses y. Another problem with uniform pricing, in comparison to second-price single-object auctions, is that producers bid strategically (with a markup).

signals and certain demand in a uniform-price auction, a comparative statics analysis of our equilibrium has, somewhat counterintuitively, a discontinuous decrease in producers' payoffs if there is a small increase in demand, such that both producers switch from being nonpivotal to pivotal with certainty.

If the auctioneer has its own signal with cost-relevant information, and if signals of the two producers and the auctioneer are affiliated, then it is beneficial for the auctioneer to disclose its signal to both producers in a discriminatory auction. Intuitively, this should also hold for a uniform-price auction, but in that case, we can only prove it when producers have independent signals. This is related to Milgrom and Weber's (1982) publicity effect for single-object auctions. Our result also concurs with Vives (2011) who shows that less informational noise makes uniform-price auctions with nonpivotal producers more competitive. Taken together, these results support the measures taken by the European Commission to increase the transparency in European wholesale electricity markets. However, disclosure of information is only beneficial up to a point. A pivotal producer can deviate to the reservation price, which ensures it a minimum profit. Moreover, in a repeated game, there is a risk that increased transparency will facilitate tacit collusion, as argued by von der Fehr (2013).

We are concerned that cost uncertainty and asymmetric information could result in significant markups in hydro-dominated electricity markets with scarce water, and that this could explain in part the extraordinarily high-price periods that typically accompany scarcity of water in such markets. Our results suggest that this problem could be reduced with improved transparency, such as public disclosure of water levels, and by disclosing a measure of the opportunity cost of water. Our results also argue in favor of market operators and regulators clearly defining and publicly disclosing contingency plans, in case of extreme system conditions. In hydro-dominated markets, improved regulatory transparency is likely to have similar procompetitive effects as improved market transparency.

These results have analogues for multiunit sales auctions, such as security auctions. In particular, given that bidders' marginal valuation of financial instruments should be approximately flat and bidders' valuations of securities typically have large common uncertainties, we believe that it could be beneficial for welfare and the auctioneer that uniform-price auctions of securities or emission permits use a bidding format that constrains offers to be flat, or to have a maximum slope. Purchase constraints in sales auctions increase the probability that both bidders are pivotal, and make them pivotal by a wider margin. This results in less competitive outcomes, at least in a one-shot game. On the other hand, purchase constraints may improve the competitiveness of secondary markets.

Appendix

We start the Appendix by proving equilibrium properties that will be useful when proving uniqueness and symmetry of BNE in auctions with discriminatory or uniform pricing. Next, we prove some relationships for conditional probabilities and conditional expected values that will be used when solving for equilibria in the two auction formats. We then prove results for the discriminatory and uniform-price auctions, and rank the two auction formats. The publicity effect is analyzed at the end of the Appendix.

□ **Uniqueness and symmetry of the equilibrium.** We first introduce the following definitions:

Definition 4.

(1) We say that $p_i(s_i)$ is sometimes price-setting if, conditional on that producer i receiving the signal $s_i \in [\underline{s}, \overline{s}]$, there is a strictly positive probability that producer i has a strictly positive output and is paid the transaction price $p_i(s_i)$.

(2) We say that firm i has an accumulation of offers at p if there is a range of signals (s_1, s_2), such that $p_i(s_i) = p$ for $s_i \in (s_1, s_2)$.

Lemma 4. Consider a BNE in a uniform-price or discriminatory auction where producer i has the strategy $p_i(s_i)$ for $s_i \in [\underline{s}, \overline{s}]$. The following equilibrium properties can be proven:

(1) Firm i cannot have a sometimes price-setting offer $p_i(s_i) \in (p_0, p_1)$ if the competitor j does not have any offer in the range (p_0, p_1) for any signal $s_j \in [\underline{s}, \overline{s}]$. Similarly, firm i cannot have a sometimes price-setting offer $p_i(s_i) \in [p_0, p_1)$ if firm j does not have any offer in the range (p_0, p_1) for any signal $s_j \in [\underline{s}, \overline{s}]$ and firm j does not have an accumulation of offers at p_0.
(2) If firm j has an accumulation of offers at p_0 for signals $s_j \in (s_1, s_2)$, then there is no signal $s_i \in [\underline{s}, \overline{s}]$ such that: $p_i(s_i) = p_0 > c_i(s_i, s_2)$.
(3) If the lowest offer that can occur for any producer in equilibrium is sometimes price-setting, then firms must have the same strategy when receiving the lowest signal, that is, $p_i(\underline{s}) = p_j(\underline{s})$.
(4) Assume that firm i has an offer $p_i(s)$ for signal s which is sometimes price-setting and such that: $p_i(s) = p_j(s)$, $p_i(\check{s}) < p_i(s)$ and $p_j(\check{s}) < p_j(s)$ for any existing $\check{s} < s$, then there is no accumulation of offers at the price $p_i(s)$.

Proof.

i) Make the contradictory assumption that the statement is true. Firm i can then increase the offer for signal s_i up to a price $p \in (p_i(s_i), p_1)$. Such a change will never change the output of producer i for the stated circumstances, but it will sometimes increase the revenue of firm i (whenever $p_i(s_i)$ is price-setting), so the deviation is strictly profitable.
ii) Make the contradictory assumption that the statement is true. Firm i can then reduce its offer price $p_i(s_i)$ by an arbitrarily small amount $\varepsilon > 0$. Due to properties of the assumed rationing rule, such a deviation will for the signal s_i increase the output of firm i by an amount that is bounded away from zero whenever the competitor receives a signal s_j in the range (s_1, s_2). The condition $p_0 > c_i(s_i, s_2) \geq c_i(s_i, s_j)$ for $s_j \leq s_2$ ensures that it is profitable for firm i to increase its output for those circumstances. Thus, the deviation is profitable for sufficiently small ε, so that any resulting reductions in the transaction price of firm i become sufficiently small.
iii) Weak-monotonicity of $p_i(s_i)$ and $p_j(s_j)$ imply that firm i has no offer below $p_i(\underline{s})$ and that firm j has no offer below $p_j(\underline{s})$. Make the contradictory assumption that $p_j(\underline{s}) \neq p_i(\underline{s})$. Without loss of generality, we assume that $p_i(\underline{s}) < p_j(\underline{s})$, where $p_i(\underline{s})$ is sometimes price-setting. However, it follows directly from i) that this is not possible in equilibrium.
iv) Make the contradictory assumption that at least one firm j has an accumulation of offers at the price $p_j(s)$ for signals $s_j \in [s, s_2]$. Without loss of generality, we assume that firm j has the (weakly) largest accumulation of offers at $p_j(s)$, that is, there is no signal $\hat{s} > s_2$ such that $p_i(\hat{s}) = p_j(s)$. It follows from ii) that $p_i(s) = p_j(s) = p_j(s_2) \leq c_i(s, s_2) = c_j(s, s_2)$, where the latter equality follows from symmetry of costs.[22] It also follows that $p_j(s_2) \leq c_j(s, s_2) < c_j(s_2, s)$, because as assumed in (3), a firm is strictly more sensitive to changes in its own signal as compared to changes in the competitor's signal. Thus, we have from (1) that $p_j(s_2) < c_j(s_2, s_i)$ for $s_i \in [s, \overline{s}]$. However, this would imply that when receiving signal s_2, firm j would have a strictly negative payoff whenever $p_j(s_2)$ is price-setting. Thus, firm j can increase its payoff by increasing $p_j(s_2)$.

■

We can use the technical results above to prove the following, which will be useful when proving uniqueness and symmetry of equilibria for both auctions.

Lemma 5. Consider an auction with uniform pricing where both producers are nonpivotal with a positive probability or a discriminatory auction. Assume that the necessary first-order conditions of offers from producers i and j have the symmetry property that $p_i'(s) = p_j'(s)$ whenever $p_i(s) = p_j(s) = p$, and there is no accumulation of offers at p. For such a first-order condition, any existing BNE in the auction must be unique and symmetric. Moreover, the unique symmetric equilibrium offer $p_i(s)$ must be invertible.

Proof. Assume that the auction has an equilibrium. The lowest offer that can occur in the equilibrium is at least partly accepted with a positive probability. We consider an auction with either discriminatory or uniform pricing. In the latter case, both producers are nonpivotal with a positive probability. Thus, the lowest offer is sometimes price-setting in the auction. Hence, it follows from 3) in Lemma 4 that $p_i(\underline{s}) = p_j(\underline{s})$. 1) and 4) ensure that there are no discontinuities in $p_i(s_i)$ at $\underline{s}$ and no accumulation of offers at $p_i(\underline{s})$. Let s^* be the highest signal in the range $[\underline{s}, \overline{s}]$, such that no producer has an accumulation of offers or a discontinuity in its offer function for $s < s^*$. Thus, the assumed symmetry property of the first-order condition and piece-wise differentiability of $p_i(s)$ and $p_j(s)$ ensure that $p_i'(s) = p_j'(s)$ for the range of signals $(\underline{s}, s^*)$. The symmetry of the initial condition $p_i(\underline{s}) = p_j(\underline{s})$ and the symmetry of slopes $p_i'(s)$ imply that $p_i(s^*) = p_j(s^*)$. Moreover, offers $p_i(s^*) = p_j(s^*)$ are not undercut with certainty by the other firm and are therefore sometimes price-setting if $s^* < \overline{s}$. Thus, we can use 1) and 4) to rule out cases where $s^* < \overline{s}$. Uniqueness follows from the assumption that $\overline{p} = c_i(\overline{s}, \overline{s})$, which ensures that $p_i(\overline{s}) = \overline{p}$ for a symmetric equilibrium, even if both producers are nonpivotal. Finally, we note that weak-monotonicity of $p(s)$ combined with no accumulation of offers implies that $p(s)$ must be piece-wise strictly monotonic, and therefore invertible for any BNE. ■

[22] Recall that we use the convention that a firm's own costs are always first in the list of signals.

Compared to discriminatory pricing, we need a stricter sufficient condition to ensure uniqueness in the uniform-price auction: both producers need to be nonpivotal with a positive probability. The reason is that an offer may sometimes be accepted in a uniform-price auction, even if it is never price-setting, as in the high-price equilibrium by von der Fehr and Harbord (1993). In the above uniqueness argument, we use Milgrom and Weber's (1982) assumption that $\overline{p} = c_i(\overline{s}, \overline{s})$. This assumption is crucial when ensuring uniqueness in an auction where both suppliers are nonpivotal with certainty, as in a single-object auction. However, if the pivotal status of suppliers is uncertain, then the uniqueness result would also hold for $\overline{p} > c_i(\overline{s}, \overline{s})$.

□ **Relationships for conditional probabilities.** Before proving the lemmas and propositions that have been presented in the main text, we will derive some results that will be used throughout these proofs. By assumption, $p_j(s_j)$ is monotonic and invertible. Thus, we get

$$\begin{aligned} \Pr\left(p_j \geq p_i \middle| s_i\right) &= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j}{\int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j} \\ \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} &= \frac{-p_j^{-1\prime}(p_i)\chi\left(s_i, p_j^{-1}(p_i)\right)}{\int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j}, \end{aligned} \tag{A1}$$

where the last result follows from Leibniz' rule. The above results and Leibniz' rule are used in the following derivations.

$$\begin{aligned} \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right] &= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i\left(s_i, s_j\right)\chi\left(s_i, s_j\right) ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j} = \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i\left(s_i, s_j\right)\chi\left(s_i, s_j\right) ds_j}{\Pr\left(p_j \geq p_i \middle| s_i\right)\int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j} \\ \frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i} &= \frac{p_j^{-1\prime}(p_i)\chi\left(s_i, p_j^{-1}(p_i)\right)\int_{p_j^{-1}(p_i)}^{\overline{s}} \left(c_i\left(s_i, s_j\right) - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\chi\left(s_i, s_j\right) ds_j}{\left(\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j\right)^2} \\ &= \frac{-\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\int_{p_j^{-1}(p_i)}^{\overline{s}} \left(c_i\left(s_i, s_j\right) - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\chi\left(s_i, s_j\right) ds_j}{\left(\Pr\left(p_j \geq p_i \middle| s_i\right)\right)^2 \int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j}. \end{aligned} \tag{A2}$$

From (A1) and (A2), we have that:

$$\begin{aligned} &-\frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i}\Pr\left(p_j \geq p_i \middle| s_i\right) - \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \\ &= \left(\frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} \left(c_i\left(s_i, s_j\right) - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\chi\left(s_i, s_j\right) ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j} - \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i\left(s_i, s_j\right)\chi\left(s_i, s_j\right) ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j}\right)\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \\ &= -\frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i\left(s_i, p_j^{-1}(p_i)\right)\chi\left(s_i, s_j\right) ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j}\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \\ &= -c_i\left(s_i, p_j^{-1}(p_i)\right)\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}. \end{aligned} \tag{A3}$$

Using the above equation, we can derive the following result:

$$\begin{aligned} &\left(1 - \frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i}\right)\Pr\left(p_j \geq p_i \middle| s_i\right) + \left(p_i - \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]\right)\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \\ &= \Pr\left(p_j \geq p_i \middle| s_i\right) + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}. \end{aligned} \tag{A4}$$

Similarly, from (A1), we have that

$$
\begin{aligned}
1-\Pr\left(p_j \geq p_i \middle| s_i\right) &= \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi\left(s_i, s_j\right) ds_j}{\int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j} \\
\mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right] &= \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} c_i\left(s_i, s_j\right) \chi\left(s_i, s_j\right) ds_j}{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi\left(s_i, s_j\right) ds_j} = \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} c_i\left(s_i, s_j\right) \chi\left(s_i, s_j\right) ds_j}{\left(1-\Pr\left(p_j \geq p_i \middle| s_i\right)\right) \int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j} \\
\frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]}{\partial p_i} &= \frac{p_j^{-1\prime}(p_i)\, \chi\left(s_i, p_j^{-1}(p_i)\right) \int_{\underline{s}}^{p_j^{-1}(p_i)} \left(c_i\left(s_i, p_j^{-1}(p_i)\right) - c_i\left(s_i, s_j\right)\right) \chi\left(s_i, s_j\right) ds_j}{\left(\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi\left(s_i, s_j\right) ds_j\right)^2} \\
&= \frac{-\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \int_{\underline{s}}^{p_j^{-1}(p_i)} \left(c_i\left(s_i, p_j^{-1}(p_i)\right) - c_i\left(s_i, s_j\right)\right) \chi\left(s_i, s_j\right) ds_j}{\left(1-\Pr\left(p_j \geq p_i \middle| s_i\right)\right)^2 \int_{\underline{s}}^{\overline{s}} \chi\left(s_i, s_j\right) ds_j}.
\end{aligned} \tag{A5}
$$

It now follows from (A5) that:

$$
\begin{aligned}
-\frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]}{\partial p_i}\left(1-\Pr\left(p_j \geq p_i \middle| s_i\right)\right) + \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right] \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \\
= \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} c_i\left(s_i, p_j^{-1}(p_i)\right).
\end{aligned} \tag{A6}
$$

□ **Discriminatory auction.** *Proof.* **(Lemma** 1) It follows from (9) that

$$
\begin{aligned}
\frac{\partial \pi_i(s_i)}{\partial p_i} = &\left(1 - \frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i}\right) \Pr\left(p_j \geq p_i \middle| s_i\right) q_H \\
&+ \left(p_i - \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \geq p_i\right]\right) \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_H \\
&+ \left(1 - \frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]}{\partial p_i}\right) \left(1-\Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L \\
&- \left(p_i - \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]\right) \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_L.
\end{aligned} \tag{A7}
$$

Using (A4) and the relation in (A6) yields:

$$
\begin{aligned}
\frac{\partial \pi_i(s_i)}{\partial p_i} = &\Pr\left(p_j \geq p_i \middle| s_i\right) q_H + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right) \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_H \\
&+ c_i\left(s_i, p_j^{-1}(p_i)\right) \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_L \\
&+ \left(1-\Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L - p_i \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_L,
\end{aligned}
$$

which gives (10). ■

The following lemma is useful when deriving results for the nonpivotal case.

Lemma 6. $e^{-\int_s^v H(u)du} > 0$ for $\underline{s} \leq s < v < \overline{s}$ and $e^{-\int_s^{\overline{s}} H(u)du} = 0$ for $\underline{s} \leq s < \overline{s}$.

Proof. It follows from (17) that

$$H(u) = \frac{\chi(u,u)}{\int_u^{\bar{s}} \chi(u,s_j)\,ds_j} = -\frac{d}{du}\ln\left(\int_u^{\bar{s}} \chi(u,s_j)\,ds_j\right) + \frac{\int_u^{\bar{s}} \chi_1(u,s_j)\,ds_j}{\int_u^{\bar{s}} \chi(u,s_j)\,ds_j}. \tag{A8}$$

The assumptions that we make for the joint probability density imply that $\frac{\int_u^{\bar{s}} \chi_1(u,s_j)ds_j}{\int_u^{\bar{s}} \chi(u,s_j)ds_j}$ is bounded. Thus, $e^{-\int_s^v H(u)du}$ is strictly positive, unless

$$e^{\left[\ln\left(\int_u^{\bar{s}} \chi(u,s_j)\,ds_j\right)\right]_s^v} = e^{\ln\left(\int_v^{\bar{s}} \chi(v,s_j)\,ds_j\right) - \ln\left(\int_s^{\bar{s}} \chi(s,s_j)\,ds_j\right)}$$

$$= \frac{\int_v^{\bar{s}} \chi(v,s_j)\,ds_j}{\int_s^{\bar{s}} \chi(s,s_j)\,ds_j}$$

is equal to zero. This is the case if and only if $\int_v^{\bar{s}} \chi(v,s_j)ds_j = 0$. It follows from the assumptions that we make on the joint probability distribution that this is the case if and only if $v = \bar{s}$. ■

Proof. **(Proposition** 1) Consider a signal $s \in (\underline{s}, \bar{s})$. Assume that $p_i(s) = p_j(s) = p(s)$ and that there is no accumulation of offers at $p(s)$. Piece-wise differentiability, weak-monotonicity of $p_j(s)$, and no accumulation of offers at $p_j(s)$ implies that $p_j(s)$ must also be piece-wise strictly monotonic in some neighborhood around s, and therefore invertible in that range. Thus, $p_j^{-1}(p_i) = s$. Hence, we get the following first-order condition from (10).

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} &= \Pr\left(p_j \geq p|\,s\right) q_H + (1 - \Pr(p \geq p|\,s))\,q_L \\ &+ (p - c_i(s,s))\,\frac{\partial \Pr\left(p_j \geq p|\,s\right)}{\partial p}(q_H - q_L) = 0.\end{aligned}$$

Using (A1) and that $p_j^{-1\prime}(p_i) = \frac{1}{p_j'(s)}$, the condition can be written as follows:

$$\int_s^{\bar{s}} \chi(s,s_j)\,ds_j q_H + \int_{\underline{s}}^{s} \chi(s,s_j)\,ds_j q_L - \frac{(p - c(s,s))}{p_j'(s)}\chi(s,s)(q_H - q_L) = 0.$$

The condition is similar for both firms. Symmetry of the underlying parameters together with $\int_s^{\bar{s}} \chi(s,s_j)ds_j q_H > 0$ and $\int_{\underline{s}}^{s} \chi(s,s_j)ds_j q_L \geq 0$, ensures that $p_j'(s) = p_i'(s)$. Thus, it follows from Lemma 5 that any existing BNE must be symmetric and unique. Below, we solve for this equilibrium.

We can use the definition in (11) to write the first-order condition on the following form:

$$p'(s) - (p - c(s,s))\,H^*(s) = 0. \tag{A9}$$

Multiplication by the integrating factor $e^{\int_s^{\bar{s}} H^*(u)du}$ yields:

$$\begin{aligned} p'(s)e^{\int_s^{\bar{s}} H^*(u)\,du} &- pH^*(s)e^{\int_s^{\bar{s}} H^*(u)\,du} \\ &= -c(s,s)\,H^*(s)e^{\int_s^{\bar{s}} H^*(u)\,du},\end{aligned}$$

so that

$$\frac{d}{ds}\left(p(s)e^{\int_s^{\bar{s}} H^*(u)\,du}\right) = -c(s,s)\,H^*(s)\,e^{\int_s^{\bar{s}} H^*(u)\,du}.$$

Next, we integrate both sides from s to $\overline{s}$.

$$\overline{p} - p(s)e^{\int_s^{\overline{s}} H^*(u)\,du} = -\int_s^{\overline{s}} c(v,v)\,H^*(v)\,e^{\int_v^{\overline{s}} H^*(u)\,du}\,dv$$

$$p(s) = \overline{p}e^{-\int_s^{\overline{s}} H^*(u)\,du} + \int_s^{\overline{s}} c(v,v)\,H^*(v)\,e^{-\int_s^{v} H^*(u)\,du}\,dv.$$

We use integration by parts to rewrite the above expression as follows:

$$p(s) = \overline{p}e^{-\int_s^{\overline{s}} H^*(u)\,du} + \left[-c(v,v)\,e^{-\int_s^{v} H^*(u)\,du}\right]_s^{\overline{s}} + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^{v} H^*(u)\,du}\,dv,$$

which gives (13), because $c(\overline{s},\overline{s}) = \overline{p}$. It is clear from (13) that $p > c(s,s)$ for $s \in [\underline{s},\overline{s})$. Hence, it follows from (A9) that $p'(s) > 0$ for $s \in [\underline{s},\overline{s})$.

It remains to show that $p(s)$ is an equilibrium. It follows from (10) and (A1) that

$$\frac{\partial \pi_i(s)}{\partial p} = \frac{\int_{p_j^{-1}(p)}^{\overline{s}} \chi\left(s,s_j\right) ds_j}{\int_{\underline{s}}^{\overline{s}} \chi\left(s,s_j\right) ds_j} q_H + \frac{\int_{\underline{s}}^{p_j^{-1}(p)} \chi\left(s,s_j\right) ds_j}{\int_{\underline{s}}^{\overline{s}} \chi\left(s,s_j\right) ds_j} q_L$$
$$- \frac{p_j^{-1\prime}(p)\,\chi\left(s,p_j^{-1}(p)\right)}{\int_{\underline{s}}^{\overline{s}} \chi\left(s,s_j\right) ds_j}\left(p - c_i\left(s,p_j^{-1}(p)\right)\right)(q_H - q_L).$$

$$\frac{\partial \pi_i(s)}{\partial p} = \frac{\chi\left(s,p_j^{-1}(p)\right)}{\int_{\underline{s}}^{\overline{s}} \chi\left(s,s_j\right) ds_j}\left(\frac{\int_{p_j^{-1}(p)}^{\overline{s}} \chi\left(s,s_j\right) ds_j}{\chi\left(s,p_j^{-1}(p)\right)} q_H + \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi\left(s,s_j\right) ds_j}{\chi\left(s,p_j^{-1}(p)\right)} q_L\right.$$
$$\left. - p_j^{-1\prime}(p)\left(p - c_i\left(s,p_j^{-1}(p)\right)\right)(q_H - q_L)\right).$$

We know that $\frac{\partial \pi_i(s)}{\partial p} = 0$ for $s = p_j^{-1}(p)$. Thus, whenever $\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j) ds_j q_H + q_L \int_{\underline{s}}^{x} \chi(s,s_j) ds_j}{\chi(s,x)}\right) \geq 0$, it follows from the above and (1) that $\frac{\partial \pi_i(s)}{\partial p} > 0$ when $s > p_j^{-1}(p) \iff p < p_j(s)$ and that $\frac{\partial \pi_i(s)}{\partial p} < 0$ when $s < p_j^{-1}(p) \iff p > p_j(s)$. Thus, $p(s)$ globally maximizes the profit of firm i for any signal s when the inequality in (12) is satisfied.

In Case 1, when costs are common knowledge, we have $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, so it follows from (13) that

$$p(s) \to \underline{c} + e^{-\int_s^{\overline{s}} H^*(u)\,du} \int_s^{\overline{s}} \frac{dc(v,v)}{dv} dv,$$

which gives (14).

For independent signals in Case 2, we have $\chi(s,s_j) = f(s)f(s_j)$, so the inequality

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi\left(s,s_j\right) ds_j q_H + q_L \int_{\underline{s}}^{x} \chi\left(s,s_j\right) ds_j}{\chi(s,x)}\right)$$
$$= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s)f\left(s_j\right) ds_j q_H + q_L \int_{\underline{s}}^{x} f(s)f(s_j) ds_j}{f(s)f(x)}\right) =$$
$$= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s_j) ds_j q_H + q_L \int_{\underline{s}}^{x} f(s_j) ds_j}{f(x)}\right) = 0 \geq 0$$

is satisfied. Moreover, we have from Definition 2 that

$$H^*(s) = \frac{f(s)(q_H - q_L)}{\int_s^{\overline{s}} f(s_j)ds_j q_H + \int_{\underline{s}}^{s} f(s_j)ds_j q_L}$$
$$= -\frac{d}{ds}\ln\left(\int_s^{\overline{s}} f(s_j)ds_j q_H + \int_{\underline{s}}^{s} f(s_j)ds_j q_L\right).$$

Thus, (13) can be written as in (15). If, in addition, the costs are insensitive to common variations in signals, so that $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, then (15) can be simplified to (16) as follows:

$$p(s) = \underline{c} + \left(\frac{q_L}{((1 - F(s))q_H + F(s)q_L)}\right)\int_s^{\overline{s}} \frac{dc(v,v)}{dv}dv$$
$$= \underline{c} + \left(\frac{q_L}{((1 - F(s))q_H + F(s)q_L)}\right)(\overline{p} - \underline{c}).$$

Both producers are nonpivotal with certainty and $q_L = 0$ in Case 3. Thus, $H^*(s)$, simplifies to (17). For affiliated signals, we have $\frac{d}{ds}(\frac{\chi(s,s_j)}{\chi(s,x)}) \geq 0$ if $s_j \geq x$, which ensures that the global second-order condition in (12) is satisfied when $q_L = 0$. If, in addition, we have that the costs are insensitive to common variations of signals, then it follows from (14) and Lemma 6 that equilibrium offers are perfectly competitive for $s < \overline{s}$. ■

Proof. **(Proposition** 2) The result follows from Definition 2, Proposition 1, and that

$$\frac{d}{dx}\left(\frac{x-y}{ax+by}\right) = \frac{y(a+b)}{(ax+by)^2} > 0$$
$$\frac{d}{dy}\left(\frac{x-y}{ax+by}\right) = -\frac{x(a+b)}{(ax+by)^2} < 0$$

if $a + b > 0$ and $x \geq y > 0$. ■

□ **Uniform-price auction.** The following derivations will be useful when analyzing uniform-price auctions. It follows from (A1) and Leibniz' rule that:

$$\mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right] = \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} (p_j(s_j) - c_i(s_i, s_j))\chi(s_i, s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i, s_j)ds_j}$$
$$= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} (p_j(s_j) - c_i(s_i, s_j))\chi(s_i, s_j)ds_j}{\Pr\left(p_j \geq p_i \middle| s_i\right)\int_{\underline{s}}^{\overline{s}} \chi(s_i, s_j)ds_j}$$

$$\frac{\partial\mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i} = \frac{-\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\int_{p_j^{-1}(p_i)}^{\overline{s}} \left(p_j(s_i) - c_i(s_i, s_j) - \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\right)\chi(s_i, s_j)ds_j}{\left(\Pr\left(p_j \geq p_i \middle| s_i\right)\right)^2\int_{\underline{s}}^{\overline{s}} \chi(s_i, s_j)ds_j}. \tag{A10}$$

Similar to (A3), it can be shown that:

$$\frac{\partial\mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i}\Pr\left(p_j \geq p_i \middle| s_i\right) + \mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right]\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}$$
$$= \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right). \tag{A11}$$

Proof. **(Lemma** 2) We have from (19) that

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} &= \frac{\partial \mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right]}{\partial p_i} \Pr\left(p_j \geq p_i \middle| s_i\right) q_H \\ &+ \mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right] \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_H \\ &+ \left(1 - \frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]}{\partial p_i}\right) \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L \\ &- \left(p_i - \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]\right) \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} q_L.\end{aligned} \tag{A12}$$

Next, we use (A6) and (A11) to simplify this expression to (20). ■

Proof. **(Proposition** 3) Note that (20) is very similar to (10) and the statements can be proven in a very similar way to the proof of Proposition 1 . In particular, it can be shown that the first-order condition is given by:

$$\begin{aligned}&\int_{\underline{s}}^{s} \chi\left(s, s_j\right) ds_j q_L - \frac{\left(p - c\left(s, s\right)\right)}{p'(s)} \chi\left(s, s\right)\left(q_H - q_L\right) = 0 \\ &p'(s) - p\hat{H}(s) = -c\left(s, s\right)\hat{H}(s).\end{aligned}$$

The property of negatively affiliated signals in (8) implies that $\frac{d}{ds}\left(\frac{\int_{\underline{s}}^{x} \chi(s, s_j) ds_j}{\chi(s,x)}\right) \geq 0$ for $x > s_j$, which is sufficient to ensure global optimality. ■

Proof. (**Proposition** 4) We let $G(P)$ be the probability that a producer's offer price is below P. This is the same as the probability that s is below $p^{-1}(P)$. Hence, it follows from (25) that

$$G\left(P\right) = \left(\frac{P - c}{\overline{p} - c}\right)^{\frac{q_L}{q_H - q_L}}.$$

From the theory of order statistics, we know that

$$G^2\left(P\right) = \left(\frac{P - c}{\overline{p} - c}\right)^{\frac{2q_L}{q_H - q_L}}$$

is the probability distribution of the highest offer price, which sets the price. Hence, the probability density of the market price is given by $2G(p)G'(p)$. Thus, the expected market price is given by:

$$\begin{aligned}\int_c^{\overline{p}} 2G\left(p\right) G'\left(p\right) p dp &= \left[G^2\left(p\right) p\right]_c^{\overline{p}} - \int_c^{\overline{p}} G^2\left(p\right) dp \\ &= \overline{p} - \left[\frac{\left(p - c\right)^{\frac{2q_L}{q_H - q_L} + 1}}{\left(\frac{2q_L}{q_H - q_L} + 1\right)\left(\overline{p} - c\right)^{\frac{2q_L}{q_H - q_L}}}\right]_c^{\overline{p}} = \overline{p} - \frac{\left(\overline{p} - c\right)\left(q_H - q_L\right)}{q_H + q_L}.\end{aligned}$$

■

Proof. **(Lemma** 3) The demand and production capacity uncertainties are independent of the signals and the cost uncertainties. Thus, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned}\pi_i(s_i) = &\ \mathbb{E}\left[p_j - \tilde{c}_i \middle| s_i, p_j \geq p_i\right] \Pr\left(p_j \geq p_i \middle| s_i\right) q_H^P \left(1 - \Pi^{NP}\right) \\ &+ \mathbb{E}\left[p_i(s_i) - \tilde{c}_i \middle| s_i, p_j \geq p_i\right] \Pr\left(p_j \geq p_i \middle| s_i\right) q_H^{NP} \Pi^{NP} \\ &+ \left(p_i(s_i) - \mathbb{E}\left[\tilde{c}_i \middle| s_i, p_j \leq p_i\right]\right)\left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L,\end{aligned} \tag{A13}$$

where

$$q_H^P = \mathbb{E}\left[\overline{q}_H \middle| \overline{q}_H < D\right].$$

It follows from differentiation of (A13) and the relations in (A4), (A6), and (A11) that:

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} &= \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right) q_H^P\left(1-\Pi^{NP}\right)\\ &+ \left(\Pr\left(p_j \geq p_i \middle| s_i\right) + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\right) q_H^{NP}\Pi^{NP}\\ &+ \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\left(c_i\left(s_i, p_j^{-1}(p_i)\right) - p_i\right) q_L\\ &+ \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L,\end{aligned} \tag{A14}$$

so

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} = \frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\left(q_H^P\left(1-\Pi^{NP}\right) + q_H^{NP}\Pi^{NP} - q_L\right)\\ + \Pr\left(p_j \geq p_i \middle| s_i\right) q_H^{NP}\Pi^{NP} + \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L,\end{aligned}$$

which can be simplified to (27), because $q^H = q_H^P(1-\Pi^{NP}) + q_H^{NP}\Pi^{NP}$. ■

Proof. **(Proposition** 5) The proof is similar to the proof of Proposition 1. ■

Proof. **(Proposition** 6) The argument is the same as in the proof of Proposition 2. ■

□ **Comparison of auction formats.** *Proof.* **(Proposition** 7) Let $\{p_H, q_H\}$ and $\{p_L, q_L\}$ be the expected transaction price and quantity for the high-output and low-output firm, respectively. Consider a symmetric equilibrium where each producer submits an offer $p(z)$ when observing the signal z. Assume that the competitors follow this equilibrium strategy, but we allow the considered producer to deviate and act as if observing a signal x, that is it makes an offer $p(x)$, although it actually observes the signal z. In this case, the expected payoff of the producer is given by:

$$\begin{aligned}\ddot{\pi}(x,z) = \left(p_H(x,z) - \mathbb{E}\left[\tilde{c}_i \middle| z, x; s_j \geq x\right]\right)\left(1 - F(x|z)\right) q_H\\ + \left(p_L(x,z) - \mathbb{E}\left[\tilde{c}_i \middle| z, x; s_j < x\right]\right) q_L F(x|z),\end{aligned}$$

where

$$F(x|z) = \frac{\displaystyle\int_{\underline{s}}^{x} \chi(z,v)\,dv}{\displaystyle\int_{\underline{s}}^{\bar{s}} \chi(z,v)\,dv}.$$

It is optimal for the producer to follow the equilibrium strategy, that is, to choose $x = z$. Hence,

$$\left.\frac{\partial \ddot{\pi}(x,z)}{\partial x}\right|_{x=z} = 0. \tag{A15}$$

Moreover, we consider equilibria such that:

$$\ddot{\pi}(\bar{s},\bar{s}) = \left(\bar{p} - \mathbb{E}\left(c\left(\bar{s}, s_j\right)\right)\right) q_L. \tag{A16}$$

In the special case where signals are independent, then we have that transaction prices and $F(x|z)$ are independent of z (for a fixed x), so

$$\begin{aligned}\frac{d\ddot{\pi}(z,z)}{dz} &= \left.\frac{\partial \ddot{\pi}(x,z)}{\partial x}\right|_{x=z} + \left.\frac{\partial \ddot{\pi}(x,z)}{\partial z}\right|_{x=z}\\ &= \left.\frac{\partial \ddot{\pi}(x,z)}{\partial z}\right|_{x=z} = -\left.\frac{\partial \mathbb{E}\left[c\left(z, s_j\right)\middle| z, x; s_j \geq x\right]}{\partial z}\right|_{x=z}\left(1 - F(z)\right) q_H\\ &\quad - \left.\frac{\partial \mathbb{E}\left[c\left(z, s_j\right)\middle| z, x; s_j < x\right]}{\partial z}\right|_{x=z} q_L F(z).\end{aligned} \tag{A17}$$

This is true for any considered auction, so it follows from (A16) and (A17) that, for any signal z, the expected equilibrium profit $\ddot{\pi}(z,z)$ must be the same in auctions with uniform and discriminatory pricing, which gives our revenue-equivalence result. Next, we consider cases where signals are affiliated. We use the superscripts P and U for pay-as-bid (discriminatory) and uniform-price auctions respectively. The low-output firm in a uniform-price auction is paid its own offer price $p(x)$, unless its output is zero. However, for the high-output firm, the price is (sometimes) set by the competitor's offer $p^U(s_j)$, and the signal of the competitor is correlated (affiliated) with z. We have that $p^U(s_j)$ is increasing with respect to its argument. Hence, it follows from Theorem 5 in Milgrom and Weber (1982) that:

$$\frac{\partial p_H^U(x,z)}{\partial z} = \frac{\partial}{\partial z}\mathbb{E}\left[p^U(u)\middle| u \geq x; x, z\right] \geq \frac{\partial p_L^U(x,z)}{\partial z} = 0.$$

Thus,

$$\begin{aligned}\left.\frac{\partial \ddot{\pi}^U(x,z)}{\partial z}\right|_{x=z} = & \left(\frac{\partial p_H^U(z,z)}{\partial z} - \left.\frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| z, x; s_j \geq x\right]}{\partial z}\right|_{x=z}\right)(1 - F(z|z))q_H \\ & - \left.\frac{\partial \mathbb{E}\left[\tilde{c}_i \middle| z, x; s_j < x\right]}{\partial z}\right|_{x=z} F(z|z)q_L \\ & - \left(p_H^U(z,z) - \mathbb{E}\left[\tilde{c}_i \middle| z, x; s_j \geq x\right]\right)q_H \left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z} \\ & + \left(p_L^U(z,z) - \mathbb{E}\left[\tilde{c}_i \middle| z, x; s_j < x\right]\right)q_L \left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z}.\end{aligned}$$

In a discriminatory auction, we have

$$p_H^P(x,z) = p_L^P(x,z) = p^P(x),$$

so

$$\frac{\partial p_H^P(x,z)}{\partial z} = \frac{\partial p_L^P(x,z)}{\partial z} = 0.$$

Production costs are the same for both auction formats. Thus, if $\ddot{\pi}^U(z,z) = \ddot{\pi}^P(z,z)$, then it follows that $p_H^U(z,z) \geq p_H^P(z,z) = p_L^P(z,z) \geq p_L^U(z,z)$. We have $\frac{\partial F(x|z)}{\partial z} \leq 0$ for affiliated signals, so whenever $\ddot{\pi}^U(z,z) = \ddot{\pi}^P(z,z)$, it must be the case that:

$$\left.\frac{\partial \ddot{\pi}^P(x,z)}{\partial z}\right|_{x=z} \leq \left.\frac{\partial \ddot{\pi}^U(x,z)}{\partial z}\right|_{x=z}. \tag{A18}$$

We know from (A15) that $\frac{\partial \ddot{\pi}(x,z)}{\partial x}|_{x=z}$ is the same for the two auction formats. Hence, it follows from the inequality in (A18) that $\frac{d\ddot{\pi}^P(z,z)}{dz} \leq \frac{d\ddot{\pi}^U(z,z)}{dz}$ whenever $\ddot{\pi}^U(z,z) = \ddot{\pi}^P(z,z)$. Thus, it follows from the boundary condition in (A16) that $\ddot{\pi}^P(z,z) \geq \ddot{\pi}^U(z,z)$. ■

□ **Publicity effect.** *Proof.* **(Proposition** 8) The second-order condition and equilibrium offers can be derived by a proof similar to the proof of Proposition 1. Similar to that proof, it can also be shown that $p^{\mathbb{C}}(s;y)$ satisfies the following first-order condition:

$$\frac{\partial p^{\mathbb{C}}(s;y)}{\partial s} = \left(p^{\mathbb{C}}(s;y) - c^{\mathbb{C}}(s,s;y)\right)H^{\mathbb{C}}(s;y).$$

Consider the same equation, but for $y_1 \geq y$. Thus, we have from $\frac{\partial H^{\mathbb{C}}(s;y)}{\partial y} \leq 0$ and Assumption 1 that whenever $p^{\mathbb{C}}(s;y) = p^{\mathbb{C}}(s;y_1)$, then $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial s} \geq \frac{\partial p^{\mathbb{C}}(s;y_1)}{\partial s}$. By assumption, $c^{\mathbb{C}}(\bar{s},\bar{s};y) = \bar{p}$, which gives the boundary condition $p^{\mathbb{C}}(\bar{s};y) = p^{\mathbb{C}}(\bar{s};y_1) = \bar{p}$, so that $p^{\mathbb{C}}(s;y_1) \geq p^{\mathbb{C}}(s;y)$ for every $y_1 \geq y$, which gives $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$. We realize that we get the same result if $\frac{\partial c^{\mathbb{C}}(s,s;y)}{\partial y}$ is sufficiently large relative to $\frac{\partial H^{\mathbb{C}}(s;y)}{\partial y}$. ■

Next, we will use an argument related to the linkage principle for single-object auctions (Milgrom and Weber, 1982). Consider an auction where the auctioneer discloses its signal y. In a symmetric equilibrium, each producer submits an offer $p^{\mathbb{C}}(z;y)$ when observing the private signal z and the common signal y. Assume that the competitors follow this equilibrium strategy, but we allow the considered producer to deviate and act as if observing a signal x, that is, it makes an offer $p^{\mathbb{C}}(x;y)$, although it actually observes the signal z. Thus, we define the following expected payments

$$W^{\mathbb{C}}(x,z) = \mathbb{E}\left[p_H^{\mathbb{C}}(x,z;y)\middle| x, z; s_j \geq x\right] \tag{A19}$$

$$L^{\mathbb{C}}(x, z) = \mathbb{E}\left[p_L^{\mathbb{C}}(x, z; y) \middle| x, z; s_j \leq x \right], \tag{A20}$$

where $p_H^{\mathbb{C}}(x, z; y)$ is the expected transaction price conditional on that the producer observes y and z, offers $p^{\mathbb{C}}(x; y)$, and gets a high output. Analogously, $p_L^{\mathbb{C}}(x, z; y)$ is the expected transaction price conditional on that the producer observes y and z, offers $p^{\mathbb{C}}(x; y)$, and gets a low output. We also find it useful to introduce

$$F(x|z) = \frac{\int_{\underline{s}}^{x} \chi(z, v)\, dv}{\int_{\underline{s}}^{\bar{s}} \chi(z, v)\, dv}.$$

We define

$$R^{\mathbb{C}}(x, z) = W^{\mathbb{C}}(x, z)(1 - F(x|z))q_H + L^{\mathbb{C}}(x, z)q_L F(x|z), \tag{A21}$$

which is the expected revenue of the considered producer when it acts as if observing a signal x. We use the superscript $\mathbb{N}$ for the case where the auctioneer does not disclose its signal y, and define

$$W^{\mathbb{N}}(x, z) = p_H^{\mathbb{N}}(x, z) \tag{A22}$$

$$L^{\mathbb{N}}(x, z) = p_L^{\mathbb{N}}(x, z), \tag{A23}$$

where $p_H^{\mathbb{N}}(x, z)$ is the expected transaction price conditional on that the producer observes z, offers $p(x)$, and gets a high output. In this case, the expected revenue can be written:

$$R^{\mathbb{N}}(x, z) = W^{\mathbb{N}}(x, z)(1 - F(x|z))q_H + L^{\mathbb{N}}(x, z)F(x|z)q_L. \tag{A24}$$

We formulate the linkage principle for these circumstances as follows:

Lemma 7. If $R^{\mathbb{C}}(\bar{s}, \bar{s}) = R^{\mathbb{N}}(\bar{s}, \bar{s}) = \bar{p}q_L$, $\frac{\partial R^{\mathbb{C}}(x, z)}{\partial z}|_{x=z} \geq \frac{\partial R^{\mathbb{N}}(x, z)}{\partial z}|_{x=z}$, and an equilibrium exists irrespective of whether the auctioneer's signal is disclosed, then the revenue of producers decreases when the auctioneer's signal is disclosed.

Proof. If the considered producer observes the private signal z and acts as if observing x, then its expected payoff is given by:

$$\begin{aligned}\ddot{\pi}^{\mathbb{C}}(x, z) = &\left(W^{\mathbb{C}}(x, z) - \mathbb{E}\left[\tilde{c}_i | x, z; s_j \geq x\right]\right)(1 - F(x|z))q_H \\ &+ \left(L^{\mathbb{C}}(x, z) - \mathbb{E}\left[\tilde{c}_i | x, z; s_j \leq x\right]\right)q_L F(x|z).\end{aligned}$$

In equilibrium, we have that it is optimal for the producer to choose $x = z$, that is,[23]

$$\begin{aligned}\left.\frac{\partial \ddot{\pi}^{\mathbb{C}}(x, z)}{\partial x}\right|_{x=z} &= W_1^{\mathbb{C}}(z, z)(1 - F(z|z))q_H - W^{\mathbb{C}}(z, z)f(z|z)q_H \\ &\quad + c(z, z)f(z|z)(q_H - q_L) + L_1^{\mathbb{C}}(z, z)F(z|z)q_L + L^{\mathbb{C}}(z, z)f(z|z)q_L \\ &= 0.\end{aligned} \tag{A25}$$

The first-order condition can equivalently be written:

$$\left.\frac{\partial R^{\mathbb{C}}(x, z)}{\partial x}\right|_{x=z} = -c(z, z)f(z|z)(q_H - q_L). \tag{A26}$$

Similar, we have for an auction where the signal y is not disclosed that:

$$\left.\frac{\partial R^{\mathbb{N}}(x, z)}{\partial x}\right|_{x=z} = -c(z, z)f(z|z)(q_H - q_L). \tag{A27}$$

[23] This is true for any y, so it is also true in expectation.

Thus, it follows from our assumptions that

$$\begin{aligned}\frac{dR^{\mathbb{C}}(z,z)}{dz} &= \left.\frac{\partial R^{\mathbb{C}}(x,z)}{\partial z}\right|_{x=z} + \left.\frac{\partial R^{\mathbb{C}}(x,z)}{\partial x}\right|_{x=z} \\ &\geq \frac{dR^{\mathbb{N}}(z,z)}{dz}.\end{aligned}$$

The result now follows from the boundary condition $R^{\mathbb{C}}(\bar{s},\bar{s}) = R^{\mathbb{N}}(\bar{s},\bar{s}) = \bar{p}q_L$. ■

Thus, if disclosing the signal y increases the linkage between producers' private signals and their revenues, then it is beneficial for the auctioneer to disclose the signal y. We use this result in the following proof.

Proof. **(Proposition** 9) We have by assumption that $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$. The signals are affiliated and both $p_H^{\mathbb{C}}(x;z;y)$ and $p_L^{\mathbb{C}}(x;z;y)$ are nondecreasing in all of their arguments, so it follows from Theorem 5 in Milgrom and Weber (1982) that $W_2^{\mathbb{C}}(x,z) \geq 0$ and $L_2^{\mathbb{C}}(x,z) \geq 0$.[24] Thus, it follows from (A21) that

$$\begin{aligned}\left.\frac{\partial R^{\mathbb{C}}(x,z)}{\partial z}\right|_{x=z} &= W_2^{\mathbb{C}}(z,z)(1-F(z|z))q_H + L_2^{\mathbb{C}}(z,z)F(z|z)q_L \\ &\quad -W^{\mathbb{C}}(z,z)\left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z} q_H + L^{\mathbb{C}}(z,z)\left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z} q_L \\ &\geq -W^{\mathbb{C}}(z,z)\left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z} q_H + L^{\mathbb{C}}(z,z)\left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z} q_L.\end{aligned} \tag{A28}$$

In an auction where the signal y is not disclosed, we have that $W_2^{\mathbb{N}}(z,z) = L_2^{\mathbb{N}}(z,z) = 0$, because the transaction price is set by the offer $p(x)$, which does not change with changed information z. Thus,

$$\left.\frac{\partial R^{\mathbb{N}}(x,z)}{\partial z}\right|_{x=z} = -W^{\mathbb{N}}(z,z)\left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z} q_H + L^{\mathbb{N}}(z,z)q_L\left.\frac{\partial F(x|z)}{\partial z}\right|_{x=z}.$$

In a discriminatory auction where the signal y is not disclosed to producers, we also have that $W^{\mathbb{N}}(z,z) = L^{\mathbb{N}}(z,z) = p(z)$. Thus, whenever we have $R^{\mathbb{N}}(z,z) = R^{\mathbb{C}}(z,z)$ at some price, then we must have $W^{\mathbb{C}}(z,z) \geq W^{\mathbb{N}}(z,z) = L^{\mathbb{N}}(z,z) \geq L^{\mathbb{C}}(z,z)$, so that

$$\left.\frac{\partial R^{\mathbb{C}}(x,z)}{\partial z}\right|_{x=z} \geq \left.\frac{\partial R^{\mathbb{N}}(x,z)}{\partial z}\right|_{x=z},$$

because $\frac{\partial F(x|z)}{\partial z} \leq 0$ when private signals of producers are affiliated. The result now follows from Lemma 7. ■

Proof. **(Proposition** 10) The second-order condition and equilibrium offers can be derived by a proof similar to the proof of Proposition 1. $\frac{\partial p^{\mathbb{C}}(s;y)}{\partial y} \geq 0$ can be shown from the same argument as in the proof of Proposition 8. ■

Proof. **(Proposition** 11) The second-order conditions in Proposition 5 and Proposition 10 are satisfied for independent signals. This ensures existence of an equilibrium, irrespective of whether the auctioneer discloses y. In the special case with independent private signals, we have that $\frac{\partial F(x|z)}{\partial z} = 0$, $W_2^{\mathbb{N}}(z,z) = L_2^{\mathbb{N}}(z,z) = 0$, so that $\frac{\partial R^{\mathbb{N}}(x,z)}{\partial z}|_{x=z} = 0$, whereas $\frac{\partial R^{\mathbb{C}}(x,z)}{\partial z}|_{x=z} \geq 0$. The result now follows from Lemma 7. ■

References

ALLEN, B. AND HELLWIG, M. "Price-Setting Firms and the Oligopolistic Foundations of Perfect Competition." *The American Economic Review*, Vol. 76 (1986), pp. 387–392.

ANDERSON, E. AND HOLMBERG, P. "Price Instability in Multi-Unit Auctions." *Journal of Economic Theory*, Vol. 175 (2018), pp. 318–341.

ANDERSON, E., HOLMBERG, P., AND PHILPOTT, A. "Mixed Strategies in Discriminatory Divisible-Good Auctions." *RAND Journal of Economics*, Vol. 44 (2013), pp. 1–32.

ANWAR, A. "Single or Multiple Pricing in Electricity Pools?" Working Paper no. 143, Department of Economics, Edinburgh University, 2006.

[24] Note that z does not have any direct effect on the offer price when x is kept fixed. However, there is an indirect effect, if z increases, then the common signal y is also likely to increase (due to its affiliation with z), and this increases offer prices in expectation.

Armantier, O. and Sbaï, E. "Estimation and Comparison of Treasury Auction Formats When Bidders Are Asymmetric." *Journal of Applied Econometrics*, Vol. 21 (2006), pp. 745–779.

——— "Comparison of Alternative Payment Mechanisms for French Treasury Auctions." *Annals of Economics and Statistics/Annales d'Économie et de Statistique*, Vol. 93/94 (2009), pp. 135–160.

Ausubel, L.M. "An Efficient Ascending-Bid Auction for Multiple Objects." *The American Economic Review*, Vol. 94 (2004), pp. 1452–1475.

Ausubel, L.M., Cramton, P., Pycia, M., Rostek, M., and Weretka, M. "Demand Reduction and Inefficiency in Multi-Unit Auctions." *Review of Economic Studies*, Vol. 81 (2014), pp. 1366–1400.

Banal-Estanol, A. and Micola, A.R. "Composition of Electricity Generation Portfolios, Pivotal Dynamics, and Market Prices." *Management Science*, Vol. 55 (2009), pp. 1813–1831.

Bartolini, L. and Cottarelli, C. "Treasury Bill Auctions: Issues and Uses." In M.I. Blejer and T. Ter-Minassian, eds., *Macroeconomic Dimensions of Public Finance: Essays in Honour of Vito Tanzi*. London: Routledge, 1997.

Beckmann, M. "Edgeworth-Bertrand Duopoly Revisited." In R. Henn, ed., *Methods of Operations Research III*. Meisenheim: Verlag Anton Hain, 1967.

Brenner, M., Galai, D., and Sade, O. "Sovereign Debt Auctions: Uniform or Discriminatory?" *Journal of Monetary Economics*, Vol. 56 (2009), pp. 267–274.

Bushnell, J., Knittel, C.R., and Wolak, F. "Estimating the Opportunities for Market Power in a Deregulated Wisconsin Electricity Market" (1999). web.stanford.edu/group/fwolak/cgi-bin/sites/default/files/BushnellKnittelWolak(002).pdf. Accessed August 31, 2018.

Deneckere, R. and Kovenock, D. "Bertrand-Edgeworth Duopoly with Unit Cost Asymmetry." *Economic Theory*, Vol. 8 (1996), pp. 1–25.

Edgeworth, F. *Papers Relating to Political Economy*. London: Macmillan, 1925.

Fabra, N., von der Fehr, N.H.M., and de Frutos, M. Á. "Market Design and Investment Incentives." *The Economic Journal*, Vol. 121 (2011), pp. 1340–1360.

Fabra, N., von der Fehr, N.-H.M. and Harbord, D. "Designing Electricity Auctions." *RAND Journal of Economics*, Vol. 37 (2006), pp. 23–46.

von der Fehr, N.H.M. "Transparency in Electricity Markets." *Economics of Energy & Environmental Policy*, Vol. 2 (2013), pp. 87–105.

von der Fehr, N-H.M. and Harbord, D. "Spot Market Competition in the UK Electricity Industry." *Economic Journal*, Vol. 103 (1993), pp. 531–546.

Gal-Or, E. "Information Transmission–Cournot and Bertrand Equilibria." *The Review of Economic Studies*, Vol. LIII (1986), pp. 85–92.

Genc, T. "Discriminatory versus Uniform-Price Electricity Auctions with Supply Function Equilibrium." *Journal of Optimization Theory and Applications*, Vol. 140 (2009), pp. 9–31.

Genc, T.S. and Reynolds, S.S. "Supply Function Equilibria with Capacity Constraints and Pivotal Suppliers." *International Journal of Industrial Organization*, Vol. 29 (2011), pp. 432–442.

Green, R.J. and Newbery, D.M. "Competition in the British Electricity Spot Market." *Journal of Political Economy*, Vol. 100 (1992), pp. 929–953.

Harsanyi, J.C. "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points." *International Journal of Game Theory*, Vol. 2 (1973), pp. 1–23.

Hästö, P. and Holmberg, P. "Some Inequalities Related to the Analysis of Electricity Auctions." *Applied Mathematics Letters*, Vol. 19 (2006), pp. 814–819.

Holmberg, P. "Supply Function Equilibria of Pay-as-Bid Auctions." *Journal of Regulatory Economics*, Vol. 36 (2009), pp. 154–177.

——— "Pro-Competitive Rationing in Multi-Unit Auctions." *Economic Journal*, Vol. 127 (2017), pp. F372–F395.

Holmberg, P. and Lazarczyk, E. "Comparison of Congestion Management Techniques: Nodal, Zonal and Discriminatory Pricing." *Energy Journal*, Vol. 36 (2015), pp. 145–166.

Holmberg, P. and Newbery, D.M. "The Supply Function Equilibrium and Its Policy Implications for Wholesale Electricity Auctions." *Utilities Policy*, Vol. 18 (2010), pp. 209–226.

Hortaçsu, A. and McAdams, D. "Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market." *Journal of Political Economy*, Vol. 118 (2010), pp. 833–865.

Hurlbut, D., Rogas, K., and Oren, S. "Hockey Stick Pricing: How the Public Utility Commission of Texas is Dealing with Potential Price Gouging." *The Electricity Journal*, Vol. 17 (2004), pp. 26–33.

Kahn, A.E., Cramton, P.C., Porter, R.H., and Tabors, R.D. "Uniform Pricing or Pay-as-Bid Pricing: A Dilemma for California and Beyond." *The Electricity Journal*, Vol. 14 (2001), pp. 70–79.

Kang, B.S., and Puller, S.L. "The Effect of Auction Format on Efficiency and Revenue in Divisible Good Auctions: A Test Using Korean Treasury Auctions." *The Journal of Industrial Economics*, Vol. 56 (2008), pp. 290–332.

Kastl, J. "On the Properties of Equilibria in Private Value Divisible Good Auctions with Constrained Bidding." *Journal of Mathematical Economics*, Vol. 48 (2012), pp. 339–352.

Klemperer, P.D. and Meyer, M.A. "Supply Function Equilibria in Oligopoly under Uncertainty." *Econometrica*, Vol. 57 (1989), pp. 1243–1277.

KREMER, I. AND NYBORG, K.G. "Divisible Good Auctions: The Role of Allocation Rules." *RAND Journal of Economics*, Vol. 35 (2004), pp. 147–159.

LEVITAN, R. AND SHUBIK, M. "Price Duopoly and Capacity Constraints." *International Economic Review*, Vol. 13 (1972), pp. 111–122.

LIZZERI, A. AND PERSICO, N. "Uniqueness and Existence of Equilibrium in Auctions with a Reserve Price." *Games and Economic Behavior*, Vol. 30 (2000), pp. 83–114.

MASKIN, E. "The Existence of Equilibrium with Price-Setting Firms." *The American Economic Review*, Vol. 76 (1986), pp. 382–386.

MCADAMS, D. "Adjustable Supply in Uniform Price Auctions: Non-Commitment as a Strategic Tool." *Economics Letters*, Vol. 95 (2007), pp. 48–53.

MILGROM, P.R. AND WEBER, R.J. "A Theory of Auctions and Competitive Bidding." *Econometrica*, Vol. 50 (1982), pp. 1089–1122.

MYERSON, R.B. "Optimal Auction Design." *Mathematics of Operations Research*, Vol. 6 (1981), pp. 58–73.

OSBORNE, M. AND PITCHIK, C. "Price Competition in a Capacity-Constrained Duopoly." *Journal of Economic Theory*, Vol. 38 (1986), pp. 238–260.

OSBORNE, M. AND RUBINSTEIN, A. *A Course in Game Theory*. Cambridge, MA: MIT Press, 1994.

PARISIO, L. AND BOSCO, B. "Market Power and the Power Market: Multi-Unit Bidding and (In)Efficiency in Electricity Auctions." *International Tax and Public Finance*, Vol. 10 (2003), pp. 377–401.

PERRY, M. AND RENY, P. J. "On the Failure of the Linkage Principle in Multi-unit Auctions." *Econometrica*, Vol. 67 (1999), pp. 895–900.

PLUM, M. "Characterization and Computation of Nash-Equilibria for Auctions with Incomplete Information." *International Journal of Game Theory*, Vol. 20 (1992), pp. 393–418.

PYCIA, M. AND WOODWARD, K. "Pay-as-Bid: Selling Divisible Goods to Uninformed Bidders." Manuscript, University of California, Los Angeles, 2015.

RILEY, J.G. AND SAMUELSON, W.F. "Optimal Auctions." *The American Economic Review*, Vol. 71 (1981), pp. 381–392.

ROSTEK, M., WERETKA, M., AND PYCIA, M. "Design of Divisible Good Markets." Manuscript, University of Wisconsin–Madison, 2010 (available at www.ssc.wisc.edu/˜mweretka/DGA.pdf).

SON, Y.S., BALDICK, R., LEE, K.H., AND SIDDIQI, S. "Short-Term Electricity Market Auction Game Analysis: Uniform and Pay-as-Bid Pricing." *IEEE Transactions on Power Systems*, Vol. 19 (2004), pp. 1990–1998.

SPULBER, D.F. "Bertrand Competition When Rivals' Costs are Unknown." *The Journal of Industrial Economics*, Vol. XLIII (1995), pp. 1–11.

TWOMEY, P., GREEN, R., NEUHOFF, K., AND NEWBERY, D. "A Review of the Monitoring of Market Power: The Possible Roles of Transmission System Operators in Monitoring for Market Power Issues in Congested Transmission Systems." *Journal of Energy Literature*, Vol. 11 (2005), pp. 3–54.

VIVES, X. "Rationing Rules and Bertrand-Edgeworth Equilibria in Large Markets." *Economics Letters*, Vol. 21 (1986), pp. 113–116.

———. "Strategic Supply Function Competition with Private Information." *Econometrica*, Vol. 79 (2011), pp. 1919–1966.

WANG, J.J.D. AND ZENDER, J.F. "Auctioning Divisible Goods." *Economic Theory*, Vol. 19 (2002), pp. 673–705.

WILSON, R. "Auctions of Shares." *Quarterly Journal of Economics*, Vol. 93 (1979), pp. 675–689.

WOLAK, F.A. "Quantifying the Supply-Side Benefits from Forward Contracting in Wholesale Electricity Markets." *Journal of Applied Econometrics*, Vol. 22 (2007), pp. 1179–1209.

———. "Report on Market Performance and Market Monitoring in the Colombian Electricity Supply Industry." Report prepared for Superintendencia de Servicios Públicos (SSPD), 2009. web.stanford.edu/group/fwolak/cgi-bin/sites/default/files/files/sspd_report_wolak_july_30.pdf. Accessed August 31, 2018.